On the Optimal Use of Fines and Imprisonment

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Abstract

This paper studies the optimal use of fines and imprisonment when wealth is heterogeneous among individuals and it may or may not be observable. When wealth is observable, for a given offender type, the fine is set to the maximum level (i.e., wealth) and the imprisonment term is either maximal or zero under general conditions. Fines and imprisonment often act as complements, meaning that maximal (zero) imprisonment is utilized for individuals with high (low) fine levels. As a result, the total sanction is often escalating with wealth. When wealth is unobservable, the optimal punishment structure changes drastically. To induce high-wealth individuals to pay higher fines and refrain from pretending to be low-wealth individuals, the total sanction must be weakly decreasing with wealth. Thus, low-wealth individuals will face higher total sanctions possibly including maximal imprisonment, while high-wealth individuals will face lower total sanctions possibly excluding imprisonment and less than maximal fines. In all cases, the inability to observe wealth lowers social welfare.

Keywords: monetary sanctions; nonmonetary sanctions; law enforcement; private information

JEL classification: D31; D62; H23; K14; K42

1. Introduction

Since Becker (1968), economists have devoted much time and energy to study the optimal public enforcement of law. Core issues include the optimal combination of sanction severity and sanction certainty and also the kinds of sanction to impose (in particular, fines and imprisonment). Becker, for example, forcefully argued that punishment should take the cheapest form, namely a fine, before resorting to more costly forms such as imprisonment. In Becker's words "fines should be used whenever feasible" (Becker, 1968, p. 193).

The social costs of punishment and, in particular, imprisonment are very high. According to the U.S. Bureau of Justice Statistics, the direct governmental cost of operating the nation's prisons, jails, and parole and probation systems is estimated at \$88.5 billions (out of a total of around \$300 billion on the justice system).[1] With more than 2.2 million people incarcerated, this sum amounts to nearly \$40,000 per detained person annually.[2] Thus, gaining a better understanding of how fines and imprisonment should be used in the public enforcement of law remains a very important policy issue.

This paper analyzes the optimal use of fine and imprisonment when individuals differ in their level of wealth, which may or may not be observable. The inability to observe wealth implies that the structure of total sanctions cannot be a simple function of wealth but has to satisfy an incentive-compatibility constraint ensuring that high-wealth individuals are deterred from mimicking low-wealth individuals. We use a standard law enforcement model (see, e.g., Polinsky and Shavell, 1984) in which the social planner chooses the structure of punishment, that is, a combination of costless fines and costly imprisonment for all wealth levels. The maximum fine is equal to the offender's level of wealth while the maximum imprisonment term is uniform for all offenders.

When wealth is observable, under general conditions, the optimal structure of punishment is as follows. First, fines are set at the maximum level (i.e, offender's level of wealth). Second, imprisonment is either set at zero or at the maximum level. In fact, we find that the often considered interior solution (i.e., a strictly positive but nonmaximal imprisonment term) stemming from a first order condition turns out to be welfare *minimizing* when the distribution of criminal gains fulfills an intuitive hazard rate condition that holds for most widely used distributions.[3] Under this condition, at low levels of imprisonment, higher imprisonment lowers social welfare because the marginal punishment cost outweighs the marginal deterrence benefits from both the act's social harm and the avoided imprisonment cost, while the ranking of marginal effects flips when the level of imprisonment surpasses a threshold. Our analysis yields a condition under which this threshold decreases with the level of wealth. The intuition runs as follows. High-wealth individuals are, at a given imprisonment term, deterred to a greater extent than low-wealth individuals due to their higher fine. The implied lower likelihood of offending among high-wealth individuals makes the use of imprisonment

[1] Some estimate the indirect costs as high as 3 times this amount. (Report). Anderson (1999) estimates an annual burden of crime at about 10 percent of GDP for the United States, attributing a large part of this aggregate burden to the operation of prisons and criminals' lost workdays. Despite its immense cost, the reliance on imprisonment is relatively high in the United States (Spamann, 2016).

[2] See also Table 1 Vera report on prison spending in 2015. "Among the 45 states that provided data (representing 1.29 million of the 1.33 million total people incarcerated in all 50 state prison systems), the total cost per inmate averaged \$33,274 and ranged from a low of \$14,780 in Alabama to a high of \$69,355 in New York".

[3] Although Kaplow (1990) highlighted the possibility that optimal imprisonment may mean either a zero or a maximum nonmonetary sanction, the internal solution has remained focal in most subsequent contributions. See our discussion in Section 2.

less socially costly when compared to low-wealth individuals.

This is a key insight for our results on the relationship between the use of imprisonment and the level of fines. Intuition suggests that imprisonment, if used at all, should be used with respect to low-wealth individuals in particular. Based on maximum fines alone, these individuals would face lower expected sanctions than high-wealth individuals and, thus, would be more likely to offend. Indeed, Becker echoes this intuition by stating that "[the] analysis implies that if some offenders could pay the fine for a given offense and others could not, the former should be punished solely by fine and the latter partly by other methods" (Becker, 1968, p. 193), which is still considered common wisdom.[4] Our analysis describes the circumstances under which this intuition holds true and, most importantly,the conditions when it is false. For many circumstances, imprisonment is imposed on high-wealth individuals but not on low-wealth ones. In other words, we find that imprisonment will often act as a complement to the level of the fine (i.e., the offender's level of wealth) and that the total sanction is increasing with wealth.

Our result that imprisonment often acts as a complement to fines seems to be at odds with what we observe in the real world. However, we can reconcile real-world observations and the results from optimal law enforcement theory by incorporating the fact that wealth is unobservable.[5] We show that the unobservability of wealth changes the structure of punishment fundamentally; in fact, it turns it upside down. The unobservability of wealth imposes a restriction on both the total sanction and its composition in terms of fines and imprisonment. The incentive-compatibility constraint means that low-wealth individuals should face (weakly) higher total sanctions than high-wealth individuals. Otherwise, high-wealth individuals will hide their assets and pretend to have low wealth. Our analysis shows that the optimal structure of punishment may take one of two different forms. First, the total, combined sanction may be uniform across all levels of wealth. This would suggest that the composition of sanctions in terms of fine and imprisonment will change with wealth from one that is dominated by imprisonment to one that is dominated by fines. Second, when the total combined sanction is not uniform across different levels of wealth, the population is split into two groups each facing the same total, combined sanction. In other words, we find there is at most one discrete drop in the level of the total sanction, at some wealth level between the minimum and the maximum. In this case, the poorer group faces a total, combined sanction which is higher than the one faced by the richer group, while within each group the mix of the sanction changes from one which is dominated by imprisonment to one that relies more on fines.

The unobservability of wealth has distributional and welfare (efficiency) implications. The former consequence results from the fact that the incentive-compatibility constraint requires that total sanctions are weakly decreasing with wealth. When compared to the punishment structure in the observability scenario, which called for a strictly increasing total sanction, the inability to observe wealth tends to worsen the position of low-wealth individuals and the improve the position of high-wealth individuals. Indeed, low-wealth individuals may suffer from harsher sanctions and from the use of imprisonment, while high-wealth individuals will

[4] Garoupa and Mungan (2019) summarize the literature stating that "The policy implication is that poor criminals ought to be imprisoned, because they cannot pay high fines, while wealthy offenders can be optimally deterred by fines (Posner, 1985; Shavell, 1985)." They also state that "When both fines and imprisonment can be conditioned on wealth, the two canonical results apply – fines should be maximal and imprisonment terms should supplement fines for poor (or less wealthy) individuals".

[5] Even if authorities could access all accounts in the offender's name, individuals may hoard cash or transfer assets to either relatives, friends, or offshore accounts.

benefit from more lenient sanctions and from the non-use of imprisonment. The welfare loss stems from the fact that the structure of punishment that is optimal when wealth is observable cannot be implemented when wealth is unobservable. Thus, any change in the structure of punishment necessitates a welfare loss.

The structure of the paper is as follows. We discuss our paper's contribution to the literature in the next section. Section 3 presents the law enforcement model we use. The optimal structure of punishment when wealth is (un)observable is laid out in Section 4. Section 6 concludes.

2. Related Literature

Our paper contributes to the literature on optimal law enforcement and in particular to papers studying the optimal use of fines and imprisonment. The three key assumptions of our analysis are (i) the social planner can utilize both a costless fine and a costly imprisonment term, (ii) potential offenders have different levels of wealth, and (iii) the level of wealth may or may not be observable.

Polinsky and Shavell (1984) is an early seminal contribution on the optimal use of fines and imprisonment. They consider the use of the instruments in isolation and in combination. Their analysis supports the arguments presented in Becker (1968) that fines should be maximal and only possibly complemented by imprisonment terms. However, when they allow for heterogeneity in wealth levels, they exclusively consider observable wealth.

For our analysis, it is important that private information about wealth reduces the set of feasible sanction structures. Levitt (1997) analyzes the role of incentive compatibility for the welfare implications of having fines as an instrument in addition to imprisonment. He considers offender types who differ in their disutility from imprisonment and their benefit from the offense, and assumes that these features are private information. In contrast, in our framework, the heterogeneity stems from wealth levels which bound feasible fines. Levitt (1997) is particularly concerned with showing that the availability of fines may have a limited welfare impact when private information about offender type prevails. Despite the different focus, the incentive compatibility highlighted by Levitt (1997) will be very important for our considerations when we assume unobservable wealth.

The paper closest to ours is Polinsky (2006). He uses the exact same set of assumptions (i)-(iii), albeit restricted to only two wealth levels while we consider the possibility of more than two wealth levels. Despite these parallels, the results presented by Polinsky (2006) stand in stark contrast to ours. When wealth is observable, he finds that low-wealth offenders face a longer imprisonment term and a higher total sanction when compared to high-wealth individuals (Polinsky, 2006, Proposition 3, (b) and (c)). This conclusion has an immediate consequence for the scenario in which wealth is unobservable. When the total sanction is decreasing with wealth, the incentive-compatibility constraint is not binding and, thus, social welfare is unaffected by the inability to observe wealth.[6] Our results contrast strongly with his and we elaborate on the reasons in Section 6.

There are other explanations in the literature for why the canonical pattern of raising the fine to its maximal level before possibly introducing imprisonment may not apply. For example, Dittmann (2006) considers the possibility that policy makers do not maximize so-

[6] Polinsky (2006) also analyzes the case where imprisonment is not used when wealth is observable, finding that the incentive-compatibility constraint changes the structure of punishment and lowers social welfare.

cial welfare but instead a convex combination of welfare and budget, and finds that this can make mandatory imprisonment optimal for the policy maker. D'Antoni and Galbiati (2007) consider the possibility that policy makers who maximize social welfare have private information about the social harm and may signal using the costly type of sanction. Garoupa and Mungan (2019) also study heterogeneous wealth levels, focusing in observable wealth. They present a rationale for not setting the fine at the level of wealth in a framework where the imprisonment term is, by assumption, independent of offender type. Chu and Jiang (1993) derive a result about the optimality of a combination of imprisonment and a less-than-maximal fine in a setting in which risk-averse individuals choose the severity of their offense knowing that the criminal gain, the level of harm, and the fine are proportional to the severity. It is apparent that these contributions are orthogonal to the present one.

Our paper also contributes to the discussion about how different law enforcement instruments are related to each other. Whereas much of the law enforcement literature views enforcement and punishment as substitutes, Garoupa (2001) shows that they may be complements when the extent of underdeterrence is substantial. In our analysis, we elaborate on the relationship of fines and imprisonment. We identify circumstances in which rational law enforcement requires that the two instruments act as complements, contradicting the intuition of the previous literature. Using data on federal fraud cases in 1984, Waldfogel (1995) presents some evidence about fines and prison terms being used as substitutes.

3. Optimal sanction structures when wealth is observable

We use a standard law enforcement model with a population of risk-neutral individuals (see, e.g., Polinsky and Shavell, 1984). Individuals differ in terms of their wealth y_i, $i = 1, ..., n$ (where a greater index represents a higher level of wealth). The private benefit from committing the offense b is randomly drawn before the individual decides whether or not to offend according to a cumulative distribution function $R(b)$ with density $r(b)$ on the support $B \subseteq [0, \infty)$; we will also refer to the hazard rate function $h(b) = r(b)(1 - R(b))^{-1}$. For simplicity, we assume there is no relationship between the distribution of benefits and the level of wealth. An offense imposes social harm h. As it is standard, we assume that $R(h) < 1$, so that some offenses are socially efficient.

If individuals undertake the offense, they face some probability of being caught and sanctioned with a fine or imprisonment or both. The socially costless fine cannot exceed the level of wealth of the offender, that is, $f_i \leq y_i$. The imprisonment term s_i, measured in units of equivalent income for the individual, cannot exceed an upper limit $\bar{s}$, which is assumed to be the same for everyone . The limit on the maximal imprisonment term can reflect physical reality (imprisonment cannot be longer than life) or moral constraints. With a cost c to the state per imprisonment unit, the total cost from sanctioning an offender with an imprisonment term s_i is $(1 + c)s_i$. In order to focus on the utilization of fines and imprisonment, we take the detection probability p as exogenous.[7] Below, we will not report fixed enforcement costs to save on notation.

In this simple setting, individuals will commit an offense if the benefits are greater than the expected punishment, that is, if $b \geq p(s_i + f_i) = p\sigma_i$, where σ_i will indicate the total cost of the fine and imprisonment to the offender. When the sanction is σ_i, the share of individuals

[7] It is reasonable to consider a fixed detection probability with respect to a specific kind of crime when the investment in enforcement effort applies to a wide range of offenses (see, e.g., Polinsky and Rubinfeld, 1991; Shavell, 1991).

with wealth y_i committing the offense will be $1 - R(p\sigma_i)$. The crime rate decreases to the same extent when the fine or the imprisonment term is increased.

In Sections 3 and 4 below, we analyze the case in which the offender's level of wealth is observable for the enforcement authority and the case in which it is not. The policy maker, who is assumed to maximize a utilitarian welfare function when choosing fines and imprisonment, can design a sanction structure that involves a total sanction $\sigma_i = f_i + s_i$ for offenders of wealth level y_i). However, when wealth is unobservable, the policy maker must ensure incentive compatibility to guarantee that offenders with wealth y_i are punished by σ_i.

When wealth is observable, the sanctions f_i and s_i can be set independently for individuals with different levels of wealth.

We indicate the social welfare associated to an individual subject to sanctions f_i and s_i as

$$W(s_i, f_i) = \int_{p(f_i+s_i)}^{\bar{b}} \big(b - h - p(1+c)s_i\big) r(b) db \tag{1}$$

Consider first the choice of the optimal fine f_i, under the constraint $f_i \leqslant y_i$. The marginal welfare effect of an increase in the fine,

$$\frac{\partial W}{\partial f_i} = p\big(h + pcs_i - pf_i\big) r(p(f_i + s_i)), \tag{2}$$

is positive as long as $f_i < h/p + cs_i$. We make the following

Assumption 1. For all i, it is $y_i < h/p$.

This implies that, for any y_i and $s_i > 0$,

$$y_i < h/p + cs_i, \tag{3}$$

so that a further increase in the fine would be socially beneficial when the fine is equal to individual wealth. This implies that it is always optimal to set the fine equal to total wealth, or $f_i = y_i$.

Next, consider the choice of imprisonment s_i, with $\leqslant s_i \leqslant \bar{s}$. The marginal welfare effect of an increase in the imprisonment term,

$$\frac{\partial W}{\partial s_i} = p\big(h + pcs_i - pf_i\big) r(p(f_i + s_i)) - p(1+c)\big[1 - R(p(f_i + s_i))\big], \tag{4}$$

consists of the benefit from deterring the marginal offender (first term) and the marginal cost from increasing the sanction on those who offend (second term). The benefit from deterring the marginal offender is $h + pcs - pf$ and thus it *increases* in the imprisonment term. In contrast, the marginal cost *decreases* because the crime rate $1 - R(p(f_i + s_i))$ decreases with the imprisonment term s_i.

In the preceding literature (see, e.g., Polinsky and Shavell, 1984; Polinsky, 2006), the optimal imprisonment term is assumed to satisfy a first-order condition for an internal solution, that is, is assumed to result from equating (4) to zero. This is adequate if the marginal welfare is positive at $s_i = 0$, then decreasing, and negative at $s_i = \bar{s}$. However, the marginal welfare effect is positive if and only if

$$\frac{r(p(f_i + s_i))}{1 - R(p(f_i + s_i))} > \frac{1+c}{h + pcs_i - pf_i}. \tag{5}$$

The right-hand side is the ratio of the total cost per-unit of imprisonment and the marginal deterrence benefit, which is decreasing in s_i. The left-hand side of (5) is the hazard rate of the distribution of benefits R. Therefore, a necessary condition for an interior solution is a decreasing hazard rate. However, the most common parametric distributions – the normal distribution and the beta and gamma distributions with bell-shaped densities – imply an increasing hazard rate.[8] As a result, we argue that the assumption of a decreasing hazard rate is less plausible and, in the following, assume instead that:

Assumption 2. The hazard rate function of the distribution of benefits from offending $h(b) = r(b)/(1 - R(b))$ is increasing in the interval $[py_1, p(y_n + \bar{s})]$.

An increasing hazard rate implies that the left-hand side in (5) is equal to the right-hand side at most once. Then, social welfare W increases or decreases everywhere with the imprisonment term, or decreases at low levels but increases at high levels. In other words, Assumption 2 implies that $W(s, y_i)$ is quasi-convex in s over the interval $[0, \bar{s}]$ for all wealth levels y_i. This is to say that an imprisonment term may solve the first-order condition, but in this case it identifies a welfare *minimum*.

We can now state necessary conditions for the use of imprisonment with respect to offenders with wealth y_i.

Proposition 1. *If the optimal total sanction includes a strictly positive imprisonment term, then*

a) the marginal welfare effect (4) *is positive at some* $s_i > 0$*;*

b) the maximum welfare level without imprisonment, $W(0, y_i)$*, is negative;*

Proof. Part *a*) follows from the fact that (4) must be positive for some $s_i > 0$ to ensure that the welfare maximum is not at $s_i = 0$. To understand part *b*), note that W tends to zero as $p\sigma_i$ tends to B. In order to have that (4) is positive at some $s_i > 0$, it must be $W(0, y_i) < 0$, otherwise we obtain a contradiction, since a positive W cannot tend to zero if $p\sigma_i$ tends to B with (4) positive at some $s_i > 0$. □

The case in which $W(0, y_i) > 0$ reflects a situation where, given the level of deterrence, offenders impose net benefits on society, that is, the benefits offenders obtain from offending is greater than the social harm they impose. In that case, it is socially undesirable to raise deterrence using imprisonment even though the marginal offender creates net losses for society. In contrast, when $W(0, y_i) < 0$, it would be desirable to achieve full deterrence to reach a welfare level of zero (no harm from violation and no punishment costs from punishing offenders).

Our analysis will be simplified, without loss of generality, by making the following assumption.

Assumption 3. For all i, the maximum sanction (maximum fine plus imprisonment) does not permit full deterrence, that is, $R(p(y_i + \bar{s})) < 1$.

We now characterize the use of imprisonment in the optimal sanction structure.

Proposition 2. *Assume that* $W(0, y_i) < 0$. *Then imprisonment is used if and only if* $W(\bar{s}, y_i) > W(0, y_i)$ *and, when used, it is used to its maximum level* $\bar{s}$.

[8] The gamma distribution implies a decreasing hazard rate only when its density is decreasing everywhere. When the density is positive and decreasing at zero, the beta distribution implies a hazard rate which is decreasing near zero and then increasing. An exponential distribution implies a constant hazard rate, while a linearly increasing hazard rate characterizes the Rayleigh distribution. The lognormal distribution produces a hazard rate that is first increasing and then decreasing, while the Pareto distribution implies a decreasing hazard rate.

Proof. The claim is a direct consequence of the fact that W is quasi-convex in imprisonment s_i. When $W(0, y_i) < 0$, since $\lim_{s\to\infty} W(s, y_i) = 0$ (with full deterrence social welfare is zero), W is either increasing everywhere, or it is first decreasing and then increasing. In the former case the solution is $s = \bar{s}$, in the latter both extreme levels may be socially optimal, depending which gives a higher level of welfare. □

The policy maker can make use of two instruments in our framework, the level of the fine and the level of imprisonment. It is important to understand how the optimal use of imprisonment depends on the level of the fine, that is, to understand whether imprisonment is a complement or a substitute to a fine. [9]. Since, in our model, fines are set at the level of wealth, our inquiry is essentially whether imprisonment should be used more often for rich individuals or poor ones.

When $W(0, y_i) < 0$, we can define

$$s^*(y_i) \equiv \text{Sup}\{s \geqslant 0 | W(s, y_i) = W(0, y_i)\} \tag{6}$$

the highest value of s (assuming s were not bounded) such that W is equal to the social welfare we have at $s_i = 0$ and $f_i = y_i$. Clearly, $s^*(y_i) = 0$ if $\partial W/\partial s_i > 0$ at all $s_i > 0$. When instead $\partial W/\partial s_i < 0$ at $s_i = 0$, since $\lim_{s\to\infty} W(s, y_i) = 0$, continuity of W implies that $s^*(y_i) > 0$. In this case, the condition $W(\bar{s}, y_i) > W(0, y_i)$ in Proposition 2 can be equivalently expressed as $s^*(y_i) < \bar{s}$.

Because imprisonment can be either zero or $\bar{s}$, we will say that imprisonment is a complement to (substitute for) a fine, if a higher fine increases the chance that imprisonment is (is not) used. Therefore, imprisonment is a complement to a fine when $s^*(y_i)$ decreases with wealth; in this case, an increase in y_i will increase the chance that $s^*(y_i) < \bar{s}$ so that imprisonment is used. Symmetrically, the chance that $s^*(y_i) > \bar{s}$ so that the optimal imprisonment is $s_i = 0$ gets higher with y_i if $s^*(y_i)$ is an increasing function of wealth.

To understand the cases formally, observe that

$$\frac{ds^*}{dy_i} = -\left(\frac{\partial W(s^*, y_i)}{\partial f_i} - \frac{\partial W(0, y_i)}{\partial f_i}\right) \times \frac{\partial W(s^*, y_i)}{\partial s_i}^{-1}, \tag{7}$$

the sign of which depends on whether $\partial W/\partial f_i$ is larger at (s^*, y_i) or at $(0, y_i)$. The imprisonment is a complement to fines if

$$\frac{ds^*}{dy_i} < 0 \iff \frac{r(py_i)}{r(p(y_i + s^*(y_i)))} < \frac{h + pcs^* - py_i}{h - py_i} \tag{8}$$

that is, whenever r is smaller – or not much greater – at py_i than at the higher level $p(y_i + s^*(y_i))$. Notice that the inequality is more likely to be satisfied the higher is c; in fact, with $c = 0$, the condition for imprisonment to complement fines requires that r is decreasing. When $c > 0$ a sufficient condition for for imprisonment to complement fine is that r is not decreasing (in fact, we can still have complementarity if r is mildly increasing). For example, if the benefits from violations are uniform, so that r is constant , then imprisonment will complement fines.

Of course, the two instruments can be substitute at some levels of wealth and complements at others.

[9]For example, Garoupa (2001) analyzes this question for enforcement and costless punishment which are commonly considered to be substitutes

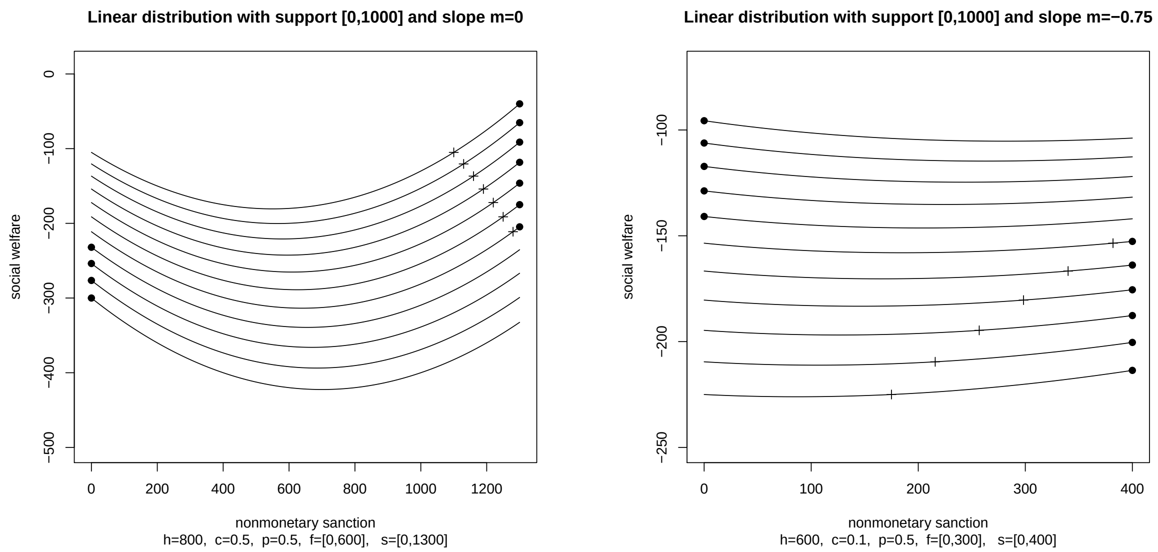


Figure 1: Complements and substitutes

The optimal sanctioning policy when wealth is observable is particularly simple to characterize when imprisonment is always a complement to or substitute for fines, i.e. when the sign of ds^*/dy does not change across the relevant interval of values of y.

Proposition 3. *Assume there exists $y^* \in [y_1, y_n]$ such that $s^*(y^*) = \bar{s}$. Then (i) if imprisonment is a complement to fines for all $y \in [y_1, y_n]$, the optimal imprisonment is zero for all $y_i \leq y^*$ and it is $\bar{s}$ for all $y_i > y^*$;*[10] *(ii) if imprisonment is a substitute for fines for all $y \in [y_1, y_n]$, the optimal imprisonment is zero for all $y^* \leq y_i$ and it is $\bar{s}$ for all $y_i < y^*$.*

Proof. Claim (i) follows from the fact that $ds^*/dy < 0$ and $s^*(y^*) = \bar{s}$ imply that $s^*(y) \lesseqgtr \bar{s}$ for all $y \gtreqless y^*$. Claim (ii) follows from the fact that $ds^*/dy > 0$ and $s^*(y^*) = \bar{s}$ imply that $s^*(y) \gtreqless \bar{s}$ for all $y \gtreqless y^*$. □

Proposition 3 thus predicts that there should be circumstances in which only rich individuals receive imprisonment terms, while poorer individuals are sanctioned only using fines. As explained in the introduction, it is commonly believed that the two instruments are substitutes, meaning that imprisonment should play a role when the deterrence based on small fines (due to small wealth levels) is insufficient. Our results cast doubt on the commonly held intuition and presents a clear criterion for when the intuition truly applies.

The two cases from Proposition 3 are illustrated by the examples in Figure 1. The curves represent social welfare W as a function of s, where higher curves correspond to higher wealth levels. The left panel assumes that criminal gains are uniformly distributed and that the cost to the state is $c = .5$. The right panel assumes that the density is linearly declining (with slope $-.75$) and that $c = .1$. The optimal imprisonment term $s \in [0, \bar{s}]$ is marked with a '•', while the critical imprisonment term allowing a welfare level similar to that without imprisonment (i.e., s^*) is marked with a '+'. The level of s^* is declining with wealth in the left panel (indicating the case of complements) and increasing with wealth in the right panel (indicating the case of substitutes).

[10] We assume here that in case of a tie, imprisonment is not utilized.

4. Optimal sanction structures when wealth is unobservable

In the literature on optimal law enforcement, it is commonly assumed that the offender's level of wealth is observable. However, in reality, it can be possible for individuals to hide their wealth from authorities. In this section, we acknowledge this fact and describe the optimal use of fines and imprisonment when *unobservable* wealth *varies* among potential offenders, assuming that the policy maker knows which wealth levels principally occur but does not know the specific wealth level of a given offender.

The policy maker can still design a sanction structure with $\sigma_i = f_i + s_i$. However, in the present circumstances, the policy maker must obey offenders' incentive compatibility constraints. We assume that offenders with wealth y_i can mimic offenders with lower wealth and have incentives to do so when any offender with lower wealth receives a smaller total sanction. In contrast, an offender with wealth y_i cannot mimic an offender with greater wealth because the latter is supposed to pay a fine in excess of y_i. Thus, the incentive compatibility constraint may be expressed by

$$\sigma_i \geqslant \sigma_j \quad \text{for any } i < j. \tag{9}$$

This implies that the total sanction for offenders with the lowest (highest) wealth, σ_1 (σ_n), is the maximum (minimum) one. Against the background of the optimal sanction structures when wealth is observable (e.g., structures where imprisonment is complementary to fines), the inability to observe wealth can radically change our conclusions.

In the rest of this section, we write the welfare obtainable from individuals with wealth y_i as

$$W_i(\sigma_i) \equiv \begin{cases} W(\sigma_i - y_i, y_i) & \sigma_i \geqslant y_i \\ W(0, \sigma_i) & \sigma_i < y_i \end{cases} \tag{10}$$

using the fact that it is always optimal to use the fine to the maximum level before resorting to imprisonment.[11]

In our characterization of optimal sanction structures when wealth is unobservable, we disregard the (uninteresting) case in which no offender type receives any imprisonment. For all other cases, the incentive compatibility condition and the escalating fine schedule implies that, if imprisonment is used for an offender type i with $y_i > y_1$, then imprisonment must also be used for the offenders with the lowest wealth. The reasoning is as follows: The total sanction of offender type 1 must be weakly higher than that for offender type i but must comprise a larger imprisonment term because $y_1 = f_1 < y_i = f_i$. In addition, we know that the total sanction for the offenders with the lowest wealth level is capped at $y_1 + \bar{s} = \bar{\sigma}$, which implies using the incentive compatibility constraint that $\sigma_i \leqslant \bar{\sigma}$.

In our characterization of optimal sanction structures, we first prove

Lemma 1. *Assume that the highest welfare from individuals with wealth y_i on the set of total sanctions defined by $\Sigma \subseteq [y_i, \bar{\sigma}]$ is attained at Sup(Σ). Then, the total sanction level Sup(Σ) also maximizes welfare from individuals with wealth y_j, $y_j < y_i$, on the set Σ.*

Proof. The function $W_j(\sigma)$ is quasi-convex on $[y_j, \bar{\sigma}]$ and $\Sigma \subseteq [y_j, \bar{\sigma}]$ because $y_i > y_j$. As a result, the maximum of $W_i(\sigma)$ on Σ is either at $\sigma_a = \text{Inf}(\Sigma)$ or at $\sigma_b = \text{Sup}(\Sigma)$. Therefore,

[11]Note, however, that the fine need not be equal to the level of wealth when enforcement authorities cannot observe the offender's wealth level. This provides a contrast relative to the optimal sanction structures when wealth is observable.

a necessary and sufficient condition for $\sigma_b \in \arg\max W_j(\sigma)$ is that $W_j(\sigma_b) > W_j(\sigma_a)$, or equivalently $W(\sigma_b - y_j, y_j) > W(\sigma_a - y_j, y_j)$. Because $W(\sigma_b - y, y) > W(\sigma_a - y, y)$ is

$$\int_{p\sigma_a}^{p\sigma_b} (h-b)r(b)db > p(1+c)\Big\{\sigma_b[1-R(\sigma_b)] - \sigma_a[1-R(\sigma_a)] + y[R(\sigma_b) - R(\sigma_a)]\Big\}, \quad (11)$$

the right-hand side is higher the higher y, as $R(\sigma_b) > R(\sigma_a)$. Therefore, if the inequality (11) is satisfied at y_i, it will be satisfied also at $y_j < y_i$. $\square$

Lemma 1 implies that the policy maker's preferences with respect to sanction structures are linked. If the use of a highest total sanction out of a set of feasible sanctions is optimal for a specific offender type i, then it is also the optimal one for all offender types with lower wealth levels. The intuition refers to the difference in punishment costs between the two total sanction levels σ_a and σ_b (given by the right hand side in (11)). The marginal change with y_j is negative as the population of offenders is larger with σ_a than with σ_b, hence a preference for σ_b over σ_a at a given wealth level must imply the same preference at the lower level.

We can now state our key result regarding the optimal sanction structure with unobservable wealth:

Proposition 4. *Let σ_i identify the optimal total sanction for individual i, so that σ_1 and σ_n indicate respectively the optimal sanction on the lowest and highest wealth individual. (i) If $\sigma_1 > \sigma_n$, then for all i such that $1 < i < n$ it will be either $\sigma_i = \sigma_1$ or $\sigma_i = \sigma_n$. (ii) If $\sigma_n < \bar{\sigma}$, then it will be $y_{i^*} \leqslant \sigma_n \leqslant y_n$, where i^* identifies the lowest wealth individual for which $\sigma_i = \sigma_n$ (i.e. $\sigma_i = \sigma_1$ for all $i < i^*$ and $\sigma_i = \sigma_n$ for all $i \geqslant i^*$).*

Proof. Consider the case the optimal sanction for individual $i < n$ implies $s_i = 0$ (no imprisonment), so that $\sigma_i = f_i \leqslant y_i$. Then, it cannot be $\sigma_i > \sigma_{i+1}$, otherwise for all $j > i$ we would have $f_j < \sigma_i$ even though setting $f_j = \sigma_i$ is feasible (because $\sigma_i < y_j$) and it is welfare improving (because W_j is increasing as long as $\sigma_j < y_j$), thus contradicting optimality. Therefore, $\sigma_i > \sigma_{i+1}$ requires $\sigma_i > y_i$. Moreover, it requires $\sigma_{i+1} \geqslant y_{i+1}$, otherwise $\sigma_{i+1} < y_{i+1}$ would once again allow to increase W_{i+1} by increasing f_{i+1}.

It follows that, for $j \leqslant i$, $W_j(\sigma_j)$ is quasi-convex in the interval $[\sigma_{i+1}, \sigma_1]$, where $s_j > 0$. As a consequence, its maximum in the interval must be either at the lower bound σ_{i+1} or at the upper bound σ_1. When $\sigma_i > \sigma_{i+1}$, W_i is maximized at σ_1, but for Lemma 1 this is also true of any other W_j with $j < i$, hence in the optimum we must have $\sigma_j = \sigma_1$ for all $j \leqslant i$.

Part (i) follows straightforwardly from the fact that $\sigma_i > \sigma_{i+1}$ implies $\sigma_i = \sigma_1$.

We now turn to Part (ii). Because σ_n is constrained only from above (by σ_{n-1}), $\sigma_n = y_n$ is always feasible. From Lemma 1 follows that when the maximum of W_n over the interval $[y_n, \bar{\sigma}]$ is $\bar{\sigma}$, such level is the maximum for all W_j over the same interval. Hence, $W_n(\bar{\sigma}) > W_n(y_n)$ implies that $\sigma_i = \bar{\sigma}$ for all i when $\sigma_n > y_n$. Therefore, $\sigma_n < \bar{\sigma}$ requires that $\sigma_n \leqslant y_n$.

Finally, if $\sigma_i > \sigma_n$ for some i, let i^* identify the lowest i for which $\sigma_i = \sigma_n$, so that $\sigma_{i^*-1} > \sigma_{i^*} = \sigma_n$. In proving Part (i) we found that $\sigma_{i^*-1} > \sigma_{i^*}$ implies $\sigma_{i^*} \geqslant y_{i^*}$, which completes the proof of Part (ii). $\square$

Part (i) of Proposition 4 reduces the maximum number of possible total sanction levels to only two whatever the number n of different wealth levels. The fact that the optimal level of the imprisonment term is either zero or $\bar{s}$ provides some intuition when seen in combination with the incentive compatibility constraint, which requires that total sanctions are weakly decreasing when maximum fines (i.e., wealth levels) are increasing. When sanctions and fines are complements in principle, imprisonment is socially valuable for offenders with high wealth levels. The incentive compatibility constraint makes it necessary that imprisonment is also used to increase the total sanction of offenders with lower wealth levels. In contrast, when sanctions and fines are substitutes, the policy maker prefers imposing imprisonment on offender types with lower wealth levels. If there is a discontinuous reduction in the level of the total sanction, imprisonment is required also at some intermediate levels of wealth in order to be able to maintain high fines for offenders with high wealth levels.

Part (ii) of Proposition 4 states that, should two different total levels of sanction be used, the total sanction for offenders with the highest wealth must not exceed their wealth level. The total sanction for offenders with the highest wealth level will include imprisonment only if it is socially desirable for this group. The incentive compatibility constraint cannot impose imprisonment on this offender type.[12]

One possibly optimal sanction structure is such that $\sigma_i = \bar{\sigma} > y_n$ for all offender types. As should be clear from Part (ii) of Proposition 4, this results only if $\bar{\sigma} > y_n + s^*(y_n)$. With $\bar{\sigma} = y_1 + s^*(y_1)$, this condition can be restated as

$$y_n - y_1 < s^*(y_1) - s^*(y_n). \tag{12}$$

This scenario thus requires that $s^*(y_1)$ is possibly substantially larger than $s^*(y_n)$, which cannot occur if imprisonment and the fine are substitutes throughout.

In contrast, if the optimal sanction structure is such that $\sigma_n < \bar{\sigma}$, it must be $W_{i^*}(\sigma_n) > W_{i^*}(\bar{\sigma})$ with i^* defining the offender type with the smallest wealth level among those offenders for whom $\bar{\sigma}$ is not an optimal sanction. When $y_i^* > y_1$, the definition of y_i^* thus implies that $W(\sigma_n; y_{i-1}^*) < W(\bar{\sigma}; y_{i-1}^*)$. These inequalities may help restrict the range of possible values of σ_n. Moreover, from $\sigma_n \geqslant y_{i^*}$ and quasi-concavity of W_i at $\sigma \geqslant y_i$ follows that $W_{i^*}(\sigma_n) > W_{i^*}(\bar{\sigma})$ implies $W_{i^*}(\bar{\sigma}) \leqslant W_{i^*}(y_{i^*})$. This helps exclude some values of i^* as incompatible with an optimal sanctioning policy.

To sum up, the optimal sanctioning policy can be characterized by the value of the total sanction σ_n, with $\sigma_n \leqslant \bar{\sigma} = y_1 + \bar{s}$. In case $\sigma_n < \bar{\sigma}$ we have

$$\sigma_i = \begin{cases} \sigma_n & \text{for } i \geqslant i^* \\ \bar{\sigma} & \text{for } i < i^*. \end{cases} \tag{13}$$

where σ_n and i^* (defined as above) solve

$$\max_{\substack{\sigma_n \leqslant y_n \\ 1 \leqslant i^* \leqslant n}} \sum_{i<i^*} \theta_i W_i(\bar{\sigma}) + \sum_{i \geqslant i^*} \theta_i W_i(\sigma_n), \tag{14}$$

with θ_i the weight of potential offenders with wealth y_i.

We used the result above to calculate the solution numerically. Assuming that $r(b)$ is uniform on $[0, B]$, we have, for $\sigma > y_i$,

$$\begin{aligned} W_i(\sigma) &= \int_{p\sigma}^{1} \left[b - h - p(1+c)(\sigma - y_i) \right] (1/B) db \\ &= \frac{1}{B} \left[\frac{1}{2} b^2 - (h + (1+c)p(\sigma - y_i)) b \right]_{p\sigma}^{B} \end{aligned} \tag{15}$$

[12] However, it must be noted that it is a possibility that y_n actually exceeds the upper bound $\bar{\sigma}$.

while, for $\sigma < y_i$,

$$W_i(\sigma) = \int_{p\sigma}^{B} \left[b - h\right] db = \frac{1}{B}\left[\frac{1}{2}b^2 - hb\right]_{p\sigma}^{B} \tag{16}$$

Figure 2 illustrates our numerical simulations. We considered a uniform distribution of b on $[0, 1000]$, $h = 800$, $c = 0.1$ and $p = 0.25$ and 11 groups of equal size/weight with wealth between 1000 and 2000. We show how the optimal solution varies considering different upper limits for the nonmonetary sanction $\bar{\sigma}$, ranging from 1000 to 2500.

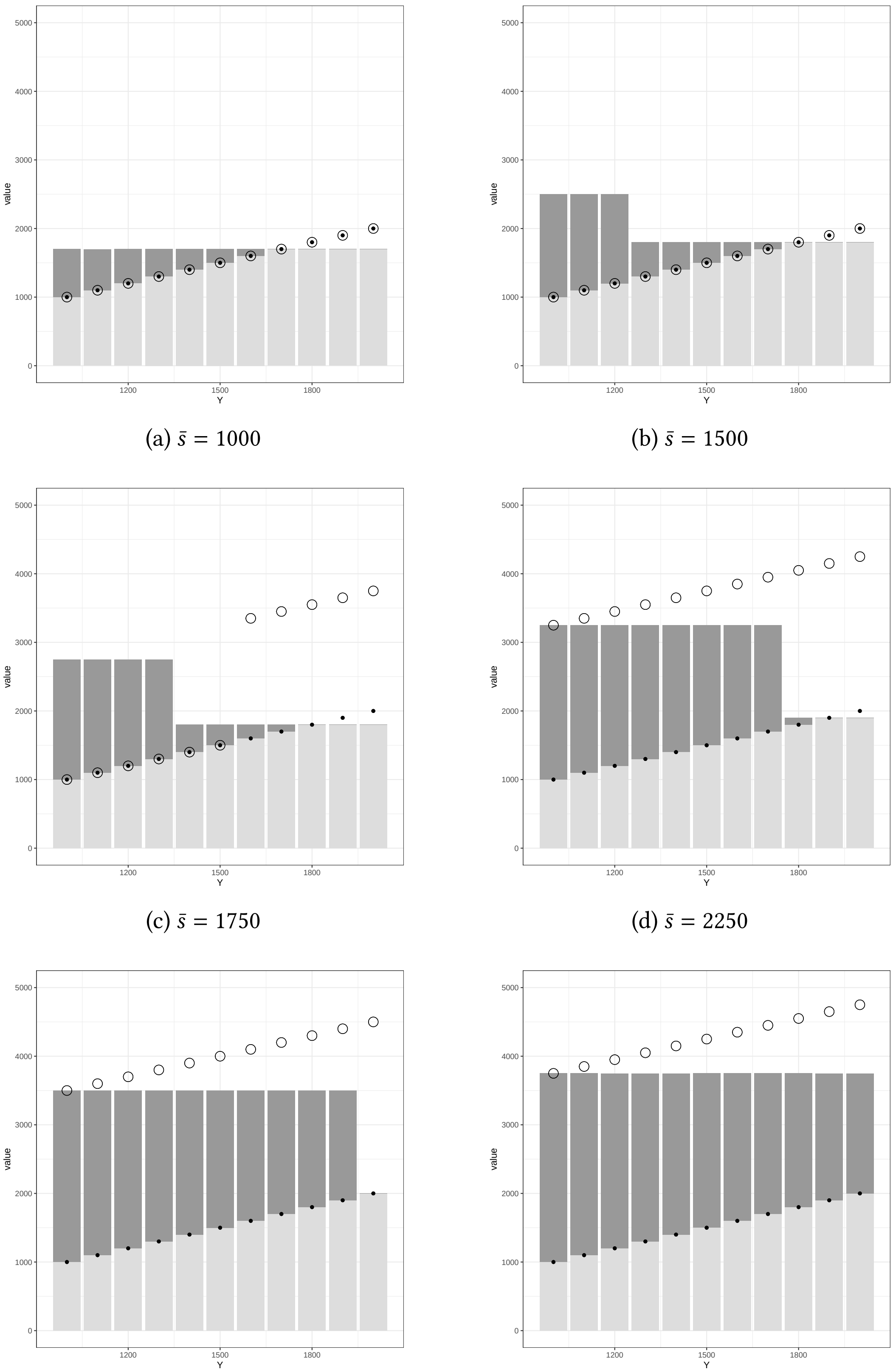
5. Welfare and distributional implications of the inability to observe wealth

The analysis by (Polinsky, 2006) delivered the surprising result that the inability to observe wealth levels is welfare-neutral when the policy maker uses a combination of fines and imprisonment in case of observable wealth. The scenario mentioned by Polinsky can result in our setting only in the extreme with $n = 2$ and fines and imprisonment as substitutes. As soon as $n = 3$, the inability to observe wealth will lower social welfare because an escalation of the total sanction (either in terms of fines alone or in terms of fines plus maximum imprisonment) cannot be included in the sanction structure when wealth is unobservable. In addition, our analysis has produced the intuition that a complementary relationship seems more relevant.

The incentive compatibility constraint requires a weakly decreasing total sanctions structure. Suppose that imprisonment and fines act as complements. When wealth is observable, offenders with the highest wealth levels receive total sanctions amounting to $y_i + \bar{s}$ and offenders with the lowest wealth levels receive total sanctions amounting to their wealth. When wealth is unobservable, offenders with the highest wealth levels cannot receive total sanctions exceeding $y_1 + \bar{s}$, with $y_1 + \bar{s} < y_i + \bar{s}$, and offenders with the lowest wealth levels in all likelihood are punished by total sanctions comprising their wealth and some imprisonment. in other words, the inability to observe wealth will in all likelihood benefit rich individuals and harm poor ones. Suppose instead that imprisonment and fines act as substitutes. This means that, when wealth is observable, offenders with the highest wealth levels receive total sanctions amounting to their wealth and offenders with the lowest wealth levels receive total sanctions amounting to $y_i + \bar{s}$. When wealth is unobservable, distributional consequences will be more complex. For example, in all likelihood at least some offender types with high wealth levels will now also face an imprisonment term whereas they did not under observable wealth. Also, it is quite likely that some offender types with low wealth levels will receive more lenient punishment as they policy maker cannot impose more than $y_1 + \bar{s}$.

6. Discussion and conclusion

Reasons for the nonneutrality of the inability to observe wealth. The inability to observe wealth lowers welfare in our setting. In contrast, Polinsky (2006) provided the surprising conclusion that it is irrelevant when both kinds of sanctions are utilized with observable wealth. What explains this striking difference? In Polinsky's framework, the optimal imprisonment for a specific offender type represents an internal solution for the optimization problem, i.e. it balances marginal benefits and marginal costs. This implies that, when the total sanction is symmetric, the marginal benefits with respect to poor offenders exceed those



(a) $\bar{s} = 1000$

(b) $\bar{s} = 1500$

(c) $\bar{s} = 1750$

(d) $\bar{s} = 2250$

(e) $\bar{s} = 2500$

(f) $\bar{s} = 2750$

Figure 2: Optimal sanctioning at different values of the maximum nonmonetary sanction

The lighter and darker bars represent respectively the monetary and nonmonetary sanction under asymmetric information; the small black dots (●) represent the levels of wealth of each group (i.e. the maximum monetary sanction), the larger white dots (◯) are the total sanctions under full information.

for rich offenders, as the greater deterrence avoids the relatively more costly total sanction on the poor offenders.[13] Thus, in Polinsky (2006), the total sanction for poor individuals exceeds that for rich individuals, implying that the incentive compatibility constraint does not bind when wealth is unobservable. In contrast, we focus on circumstances in which corner solutions are optimal for imprisonment, thereby developing the discussion in Kaplow (1990). Whereas Polinsky (2006) does not anticipate a role for maximal imprisonment sentences, we show that, if the distribution function fulfils a nonrestrictive condition, optimal imprisonment will be either zero and maximal.[14] As a result, if fines and imprisonment act as complements, the incentive compatibility constraint excludes that the sanction structure that is optimal with observable wealth can be implemented with unobservable wealth. Indeed, we show that sanction structures with unobservable wealth may look very different from those prescribed with observable wealth, as the former use at most two levels for the total sanction and produce the tendency that the bulk of imprisonment terms is concentrated on poor individuals.

Implications of sanctioning costs increasing with wealth. Following Polinsky (2006), we assumed that all potential offenders suffer the same punishment cost from a given prison term. Famously, Lott (1987) has argued that the opportunity costs of wealthy individuals exceed those of less well-off individuals.[15] At the intuitive level, greater per-unit cost makes imprisonment more effective in terms of deterrence. At the same time, greater per-unit cost also mean greater cost of using this law enforcement instrument. The critical sanction level $s^*(y_i)$ and how it responds to a change in wealth y_i is critical for whether the policy maker concentrates imprisonment rather on rich than poor individuals. When we consider a punishment cost a_i that is increasing with wealth, we find that $s^*(y_i)$ is more likely to be decreasing with wealth when compared to the benchmark case where punishment cost is independent of wealth (see Appendix A for a formal proof). In other words, sanctioning costs increasing with wealth tilt the overall ambiguous relationship between prison terms and fines towards complementarity; this makes the case that the inability to observe wealth implies a change in the sanctioning structure and a welfare losses more likely—that is to say, it reinforces our conclusion.

Concluding remarks. Public law enforcement may use fines and/or imprisonment terms to deter potential offenders from socially harmful activities. Because fines are often substantially constrained by wealth levels and imprisonment represents a very costly instrument, gaining an understanding of the optimal joint use of the two sanctions is important for policy making. In this paper, we highlight that corner solutions seem particularly important with respect to prison terms, that surprisingly simple sanction structures result as socially optimal when wealth is unobservable (sometimes approximating equal treatment before the law), and that the inability to observe wealth is detrimental to welfare.

[13] This relatively greater cost results from the fact that the total sanction of poor individuals comprises a greater imprisonment term.

[14] As we argued above, most commonly used distribution functions induce such corner solutions.

[15] A similar assumption is used in Polinsky and Shavell (1984).

Appendix

A. Variability in sanctioning costs

To explore the implications from the association of wealth and imprisonment costs, we assume that the level of the fine is set at the maximum level and that the per-unit imprisonment cost is increasing in the individual's wealth. This uses our result about the socially optimal fine and follows the ideas laid out in Lott (1987) and Polinsky and Shavell (1984), for example. We amend (1) to

$$W(s_i, f_i, a_i) = \int_{p(f_i + a_i s_i)}^{\bar{b}} \big(b - h - p(a_i + c)s_i\big) r(b) db \tag{17}$$

where $a_i = a(y_i)$ with $a' > 0$. The effect of an increase in y_i on s^* is now given by:

$$\frac{ds^*}{dy_i} = -\left(\frac{\partial W(s^*, y_i, a_i)}{\partial f_i} + \frac{\partial W(s^*, y_i, a_i)}{\partial a_i} a'(y_i) - \frac{\partial W(0, y_i, a_i)}{\partial f_i}\right) \times \frac{\partial W(s^*, y_i, a_i)}{\partial s_i}^{-1}, \tag{18}$$

which modifies (7) to account for the presence of a_i. Notably, $\partial W(s_i, y_i, a_i)/\partial f_i$ does not depend on a_i, so that its expression is still given by (2). Therefore, the numerators in (7) and (18) only differ for the term

$$\frac{\partial W(s^*, y_i, a_i)}{\partial a_i} a'(y_i) = s^* \Big[(h + pcs^* - py_i) r(b^*) - (1 - R(b^*))\Big] a'(y_i) \tag{19}$$

where $b^* = p(y_i + as^*)$. Because the denominator

$$\frac{\partial W(s^*, y_i, a_i)}{\partial s_i} = a_i (h + pcs^* - py_i) r(b^*) - (a_i + c)(1 - R(b^*)) \tag{20}$$

must be positive for W to be increasing in s_i at s^*, it follows that, with $c \geqslant 0$, the term (19) is positive too. We conclude that (18) is more likely than (7) to take a negative value. In other words, when the individual cost of the nonmonetary sanction a_i is increasing in y_i, we expect that monetary and nonmonetary sanctions are more likely to be complementary.

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