

# The Complementary Role of Distributive and Criminal Equity

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## Abstract

We provide a theoretical examination on how the distribution of wealth in society affects the social costs of crime and law enforcement. We show that a reduction in inequality reduces these costs when enforcement and non-monetary punishment are equitable, i.e., they do not discriminate among offenders based on their wealth. However, when enforcement or non-monetary punishment is discriminatory, a reduction in inequality may increase the social costs of crime and law enforcement, in particular when it occurs among poorer individuals. Thus, there is a complementarity between equity in criminal justice and distributional equity.

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## 1. Introduction

The link between inequality and compliance with the norms has been the subject of much economic analysis.

The tradition of economic analysis of crime, started by [Becker \(1968\)](#), has always emphasized that a higher level of inequality, implying lower returns from legal activities, increases the relative reward of the poorer individuals for crime (see also [Ehrlich, 1973](#)).

While the literature mostly emphasized the “demand side” story of how income or wealth affect the benefit from violations (e.g. [Benoit and Osborne, 1995](#); [Demougin and Schwager, 2000](#)), we focus instead on the relatively less explored “supply side”, concerning the cost of enforcement and violations. In particular, we examine the link between distribution of wealth and the social cost of crime and law enforcement, analyzing under what conditions more wealth inequality is conducive to a lower social cost of law enforcement.

The key insight is that, when sanctions are limited by wealth, redistributing wealth from richer to poorer individuals decreases the social costs of law enforcement, because (1) the level of deterrence increases if wealth is moved to wealth constrained (i.e. poorer) individuals; (2) even when aggregate deterrence is unaffected, increasing deterrence of poorer individuals is socially more beneficial; this is because poorer individuals, who are subject to lower sanctions, tend to offend when their benefit from violation is lower, thus generating a higher net social loss.

A few remarks on our setting and how it may differ from other analyses are necessary.

In order to isolate the effect of the supply side, represented by sanctioning policies, we simplify the setting so that any direct effect of wealth on the distribution of benefits from violation is left out of our analysis. First of all, we consider that such benefits are monetary. This is the case, for example, with violations of norms and rules that are designed to prevent harmful activities or externalities and impose costly actions or precautions to the individual.. Namely, we think of the violation of rules requiring that the individual spends a certain amount of money to avoid a possible social harm.<sup>1</sup> Moreover, we consider that the cost of a given sanction is the same for all individuals, regardless of their wealth. This is a natural assumption when sanctions are monetary. With nonmonetary sanction, this assumption requires additional discussion, which we defer to a later section.

With regard to nonmonetary sanctions, we follow the standard account by considering that such sanctions are used only as a supplement to monetary sanction when the latter

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<sup>1</sup>Notice that a common argument which applies to criminal activities is that an individual allocates his time between legal and illegal activities, and the former are less remunerative for lower income individuals, who will have a higher propensity to engage in crime. However, if we consider that avoiding a violation is time spending, we would obtain the opposite conclusion that compliance is more costly to an higher income individual. Our assumption is aimed at avoiding to discuss these different cases.

have been used at their maximum level.

Things are kept simple also by assuming that individuals are endowed with wealth, and no production of income is taken into account, and they are risk neutral.

This paper extends and develops an argument presented in [Tabbach \(2012\)](#) (which, to the best of our knowledge is the only previous paper that focuses on the relation between wealth distribution and enforcement). In that paper, however, the analysis was limited to the case of only two levels of wealth and of monetary sanctions alone, and the characterisation of redistribution was straightforward. We consider here a richer set of sanctioning policy, which includes nonmonetary sanctions, and inquire a more general question: given any two distributions of wealth, such that the first is less equal than the second, how does the social cost of enforcement change as we move from the former to the latter?

To this aim, and in order to exclude any prior value judgement on the optimal level of inequality, we measure the social cost as the aggregate money equivalent value of enforcing a norm. In other words, we assume a social welfare function which is neutral to the distribution of wealth. Preference for a more equitable wealth distribution, when it arises, is a consequence of minimization of social cost of law enforcement.

An interesting finding of our analysis is that the link between minimization of the cost of enforcement and wealth is clear cut only when law enforcement is itself equitable, in the sense that enforcement and statutory sanctions cannot discriminate among offenders based on their wealth. We find that, when law enforcement is instead inequitable, more wealth inequality may reduce the social cost of enforcement. This result can be interpreted as indicating that there is a complementarity between equity in criminal justice and distributional equity.

The paper proceeds as follows. Section 2 analyzes the benchmark case where enforcement is non-discriminatory and sanctions are in the form of fines. In section 3 we analyze the effects of allowing for non-monetary sanctions, contrasting the cases in which sanctions can and cannot be discriminatory. In section 4 we analyze the consequences of relaxing the assumption that enforcement effort is the same for all offenders, regardless of their wealth.

## **2. Monetary sanctions**

Our baseline model of enforcement in a society with individuals characterised by different levels of wealth considers only monetary sanctions and follows [Polinsky and Shavell \(1991\)](#). Risk-neutral individuals contemplate whether to commit a harmful act. The benefits to individuals from committing the act ( $b$ ) is private information, distributed according to a cumulative distribution function  $R(b)$  with density  $r(b)$  defined on the interval  $[0, \infty)$ . The harmful act imposes harm  $h > 0$  on society at large. To control

the behavior of individuals, the social planner can impose a monetary sanction  $f$  on individuals. Effectively this monetary sanction, or fine, cannot exceed the wealth of an individual ( $w$ ). Thus for a given “statutory” fine  $\hat{f}$ , the effective fine paid by an individual of wealth  $w$  will be the minimum between  $\hat{f}$  and  $w$ . Wealth in society is distributed according to a cumulative distribution function  $G(w)$ , with  $G(0) = 0$ , possibly such that  $G(\bar{w}) = 1$  for some finite  $\bar{w}$  if wealth is bounded. To emphasize the supply-side effect of wealth distribution on the social cost of crime and law enforcement, leaving aside the possibility that wealth directly affects the benefit from violations, we will assume throughout the paper that  $b$  and  $w$  are independently distributed.

In addition to the fine, the social planner sets the probability of punishment  $p$ . In this section we assume that  $p$  must be common to all individuals regardless of their wealth; in other words, we assume that  $p$  cannot depend on the wealth of an individual. This can be justified, for example, if the criminal justice system is equitable. We shall relax this assumption in section 4 where we allow for discriminatory enforcement. The cost of enforcement is given by an increasing and convex function  $c(p)$ , with  $c' > 0$  and  $c'' > 0$ . Fines are socially costless.

### 2.1. Optimal sanctioning policy

Given the distribution of wealth in society, the social planner’s problem is to choose  $p$  and the statutory fine  $\hat{f}$  to minimize the social costs, given by the sum of the costs from the harmful act (net of the offenders’ benefit) and the enforcement costs:

$$\int_0^{\bar{w}} \left[ \int_{pf(w)}^{\infty} (h - b)r(b)db \right] dG(w) + c(p). \quad (1)$$

where  $f(w) = \min\{\hat{f}, w\}$ .

The optimal sanctioning policy, characterized by [Polinsky and Shavell \(1991\)](#), involves that:

- the optimal level of the statutory fine is  $\hat{f} = h/p$ , so that the optimal effective fine is  $f^*(w, p) = \min\{h/p, w\}$ ;<sup>2</sup>
- the optimal  $p$ , assuming it is positive and less than 1, is such that either  $h/p \geq \bar{w}$ , so that the effective fine is equal to  $w$  for all individuals, or  $h/p < \bar{w}$ , so that for some individuals, namely those with a level of wealth higher than  $h/p$ , the effective fine

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<sup>2</sup>For a given  $p$ , the optimal fine  $f$  at wealth  $w$  is obtained by minimizing  $\int_{pf}^{\infty} (h - b)dR(b) + c(p)$  under the constraint  $f \leq w$ . Because the derivative with respect to  $f$  is  $-p(h - pf)r(pf)$ , whose sign depends on  $h - pf$ , the optimal fine is the lowest value between  $w$  and  $h/p$ .

is less than their wealth;<sup>3</sup>

- Individuals whose wealth is less than  $h/p$  will be underdeterred, in the sense that they will violate even if  $b < h$ , while all individuals with wealth higher than or equal to  $h/p$  will be efficiently deterred (they will violate if and only if  $b > h$ ).

The planner's optimization problem can be conveniently restated as a two-step process. First, we define

$$\Psi(f, p) = \int_{pf}^{\infty} (h - b)r(b)db \quad (2)$$

which represents the net cost of violations and sanctions as a function of the sanction actually paid  $f$  and the probability  $p$ .<sup>4</sup> For given  $p$  and  $w$ , the optimal fine  $f^*(w, p) = \min\{h/p, w\}$  minimizes  $\Psi$ . Taking into account the distribution of wealth, the optimal probability  $p$  is found by minimizing the social cost, now written as:

$$\int_0^{\bar{w}} \Psi(f^*(w, p), p)dG(w) + c(p). \quad (3)$$

## 2.2. Inequality and the social cost of enforcement

To analyse the effect of wealth inequality on the (optimal) social cost of crime and law enforcement as given by (1), we need to clarify what we mean by higher or lower inequality.

To this purpose, consider two distributions  $G$  and  $\tilde{G}$ , with support  $[0, \bar{w}]$ ,<sup>5</sup> and having the same mean. According to a common definition, the distribution  $\tilde{G}$  is more equal than the distribution  $G$  if the former can be obtained from the latter by a sequence of equalizing transfers, such as Pigou-Dalton transfers.

As is well known (Rothschild and Stiglitz, 1970), such notion of  $\tilde{G}$  being more equal than  $G$  is equivalent to the requirement that, for any (strictly) convex and decreasing  $\phi$ ,

$$\int_0^{\bar{w}} \phi(w)dG(w) > \int_0^{\bar{w}} \phi(w)d\tilde{G}(w). \quad (4)$$

This, on turn, amounts to an “integral condition” which, for distributions having the same mean, can be interpreted in terms of the Lorenz curve. Namely, condition (4) is a necessary and sufficient condition for the Lorenz curve corresponding to  $G$  to always lie above the Lorenz curve corresponding to  $\hat{G}$  (Atkinson, 1970).

<sup>3</sup>The optimal  $p$  is usually assumed to be an interior solution, obtained from the first order condition:

$$\int_0^{f^*(\bar{w}, p)} w(pw - h)r(pw)dG(w) + c'(p) = 0.$$

<sup>4</sup>Observe that such cost does not depend directly on  $w$ , as wealth affects deterrence and violations only inasmuch as it constrains the fine  $f$ .

<sup>5</sup>Hence,  $G(0) = \tilde{G}(0) = 0$  and  $G(\bar{w}) = \tilde{G}(\bar{w}) = 1$ .

Given the similarity between (4) and (3), this result allows us to derive conclusions on the effects of changes in wealth distribution based on the shape of  $\Psi$ , defined by (2) in the case of monetary sanctions.

Since  $\Psi$  is affected also by the distribution of benefits  $R(b)$ , we need to rule out cases where the result is driven by made-to-measure wealth transfers exploiting specific features of such distribution, as might be the case, for example, when  $r(b)$  presents sudden changes at specific values or intervals of  $b$ . To this purpose, we impose some “regularity” condition and require that  $r(b)$  is smooth and does not increase sharply as  $b$  increases:

**Assumption 1.** *The density  $r$  describing the distribution of benefits is assumed to be differentiable and satisfies the following restriction (on its elasticity)*

$$\frac{br'(b)}{r(b)} < \frac{b}{h-b} \quad 0 \leq b \leq h. \quad (5)$$

The right hand side is increasing in  $b$ , making the condition less restrictive as the benefit increases. Distributions satisfying this assumption include the cases of density functions that are strictly decreasing or constant for  $b < h$ .

We can now state:

**Proposition 1.** *Assume that, in a population of individual with varying wealth, the social planner applies an optimal sanctioning policy characterized by a common probability  $p$  and maximal fine  $\hat{f} = h/p$ . Then, holding Assumption 1, any set of equalizing transfers of wealth affecting some individuals with wealth  $w < \hat{f}$  reduces the social costs of law enforcement.*

*Proof.* From (2) we have:

$$\Psi_f = -p(h - pf)r(pf) < 0 \quad (6)$$

$$\Psi_{ff} = p^2[r(pf) - (h - pf)r'(pf)] > 0 \quad (7)$$

where  $\Psi_{ff} > 0$  (which implies the convexity of  $\Psi$  in  $f$ ) follows from Assumption 1. It follows that  $\Psi(f^*(w, p), p)$  is convex and decreasing in  $w$  when  $f^*(w, p)$  is an increasing and concave function of  $w$ , that is when  $w < h/p$ , while it is constant in  $w$  when  $f^*(w, p)$  is also constant, i.e. for  $w \geq h/p$ .

Consider a different wealth distribution  $\tilde{G}$ , more equal than  $G$ , and let  $\tilde{p}$  be the level of enforcement that minimizes the social cost (3) when the distribution is  $\tilde{G}$ . For any  $p$ ,

we have

$$\int_0^{\bar{w}} \Psi(f^*(w, p), p) dG(w) - c(p) \geq \int_0^{\bar{w}} \Psi(f^*(w, p), p) d\tilde{G}(w) - c(p) \geq \int_0^{\bar{w}} \Psi(f^*(w, \tilde{p}), \tilde{p}) d\tilde{G}(w) - c(\tilde{p}). \quad (8)$$

where the first inequality follows from the fact that  $\tilde{G}$  is more equal than  $G$  and  $\Psi$  is concave and decreasing in  $w$ , while the second inequality follows from the fact that  $\tilde{p}$  minimizes the social cost when the distribution is  $\tilde{G}$ .

The specification that the change in distribution must involve individuals with wealth  $w < \hat{f}$  is necessary to exclude the case that  $G$  and  $\tilde{G}$  differ only for  $w \geq h/p$ , in which case passing from  $G$  to  $\tilde{G}$  will affect neither  $f$  nor  $\Psi$ , so that the social cost will be unchanged. In fact, when  $G(w) \neq \tilde{G}(w)$  for some  $w < h/p$ , the first inequality in (14) is strict and the social cost is strictly lower with  $\tilde{G}$ .  $\square$

Proposition 1 reflects first of all the circumstance that some (richer) individuals may not be wealth constrained, hence by transferring part of their wealth to less rich individuals who are wealth constrained it is possible to increase deterrence of the latter without reducing deterrence of the former. However, there is a second, more general and possibly less obvious, effect of wealth redistribution on the social cost, which applies even when overall deterrence is unaffected: the net social cost of a violation, given by  $h - b = h - pf(w)$ , is higher the lower is the expected sanction, which depends on wealth.

**Corollary 1.** *The social cost of crime and enforcement is minimized when wealth is perfectly equalized.*

*Proof.* It follows from the fact that the perfectly egalitarian distribution can be obtained from any other distribution by a set of equalizing transfers.  $\square$

### 3. Non-monetary sanctions

In the previous section we analyzed the case in which only monetary sanctions are utilized. However, the fact that some individuals are underdeterred because of their inability to pay the monetary sanction in full due to their limited wealth provides a justification for using non-monetary sanctions as well. Because non-monetary sanctions are more costly from a social perspective, as they impose a cost on the individual and also on society, the law and economics literature argues that they should be employed only when monetary sanctions are used to their maximum extent (Becker, 1968; Polinsky and Shavell, 1984).

When wealth is observable, the optimal non-monetary sanction, at least in principle, could depend on the wealth of offenders. Nevertheless, the notion that non-monetary

sanction can vary among offenders according to their wealth has been challenged by some scholars on both practical and moral grounds. Accordingly, [Garoupa and Mungan \(2019\)](#) analyse the optimal sanctioning policy assuming that the social planner cannot discriminate among offenders, so the same additional non-monetary sanction  $s$  must apply to all offenders irrespective of their wealth. [Polinsky \(2006\)](#), on the other hand, assumes that the non-monetary sanctions need not be uniform to all offenders. We will analyze the effects of redistribution under these alternative scenarios.

To proceed in our analysis, the function  $\Psi$  defined in (2) must be amended to take into account the presence of nonmonetary sanctions:<sup>6</sup>

$$\Psi(f, s, p) = \int_{p(f+s)}^{\infty} (h + p(1 + \gamma)s - b)r(b)db; \quad (9)$$

where  $s \geq 0$  is the nonmonetary sanction, or more precisely, its monetary cost to the offender;  $\gamma$  is the unitary cost of the nonmonetary sanction for society at large.

### 3.1. *Non-discriminatory non-monetary sanctions*

When the non-monetary sanction imposed by the planner is the same for all offenders irrespective of their wealth, its optimal value depends on the distribution of wealth. Given the optimal fine  $f^*(w, s, p)$ , which minimizes (9) under the constraint  $f \leq w$ , the optimal  $s$  and  $p$  solve

$$\min_{\substack{s \geq 0 \\ 0 \leq p \leq 1}} \int_0^{\bar{w}} \Psi(f^*(w, s, p), s, p)dG(w) - c(p). \quad (10)$$

The optimal sanctioning policy has been characterized by [Garoupa and Mungan \(2019, Proposition 1\)](#) as follows:

- the optimal statutory fine is  $\hat{f} = h/p + \gamma s$ , as the cost to society of a violation now includes the cost of the non-monetary sanction; as a result,  $f^*(w, s, p) = \min\{h/p + \gamma s, w\}$ ;
- some individuals with  $w < \hat{f}$  will be underdeterred, in the sense specified in the previous section;
- in the optimum we may have  $s > 0$  even for individuals for whom  $w > \hat{f}$ , i.e. the optimal sanctioning policy may require that high wealth individuals are subject to non-monetary sanctions even if they could still pay a fine higher than  $\hat{f}$ .

Notably, the availability of a non-monetary sanction supplementing the fine may reduce reliance on the latter, but it does not modify the characteristics of the optimal sanctioning policy.

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<sup>6</sup>We will abuse notation by using the same symbol  $\Psi$  to indicate both the function defined in (2) and in (9).



We amend Assumption 1, by stating

**Assumption 1'.** *The density  $r$  describing the distribution of benefits is assumed to be differentiable and satisfies the following restriction (on its elasticity)*

$$\frac{br'(b)}{r(b)} < \frac{b}{h + (1 + \gamma)ps - b} \quad ps \leq b \leq h + (1 + \gamma)ps \quad (11)$$

This assumption on the shape of  $r$  is more restrictive than Assumption 1, due to the presence of the term  $(1 + \gamma)ps$  at the denominator of the right hand side. Yet, the right hand side is still positive and increasing in  $b$ .

Proposition 1 can be straightforwardly extended to this case.

**Proposition 2.** *Assume that, in a population of individual with varying wealth, the social planner applies an optimal sanctioning policy characterized by a common probability  $p$ , a common non-monetary sanction  $s \geq 0$  and maximal fine  $\hat{f} = h/p + \gamma s$ . Then, holding Assumption 1', any set of equalizing transfers of wealth affecting some individuals with wealth  $w < \hat{f}$  reduces the social costs of law enforcement.*

*Proof.* The proof is similar to the proof of Proposition 1. Taking the partial derivatives of (9) we have:

$$\Psi_f = -p(h + p\gamma s - pf)r(p(f + s)) < 0 \quad (12)$$

$$\Psi_{ff} = p^2[r(p(f + s)) - (h + p\gamma s - pf)r'(p(f + s))] > 0 \quad (13)$$

where  $\Psi_{ff} > 0$  follows from Assumption 1'.

To make notation more compact, we define  $\Phi(w, s, p) \equiv \Psi(f^*(w, s, p), s, p)$ . With  $f^*(w, s, p) = \min\{\hat{f}, w\}$  an increasing a concave function of  $w$ , we conclude that  $\Phi(w, s, p)$  is a nondecreasing and convex function of  $w$  (namely,  $\Phi$  is strictly decreasing and strictly concave in  $w$  for  $w < \hat{f}$  and constant in  $w$  for  $w \geq \hat{f}$ ).

Consider a different wealth distribution  $\tilde{G}$ , more equal than  $G$  and let  $\tilde{s}$  and  $\tilde{p}$  be respectively the nonmonetary sanction and the level of enforcement that minimizes the social cost (10) when the distribution is  $\tilde{G}$ . For any  $s$  and  $p$  (including the values that minimize the social cost when the distribution is  $G$ ), the social cost of enforcement satisfies:

$$\int_0^{\bar{w}} \Phi(w, s, p) dG(w) - c(p) \geq \int_0^{\bar{w}} \Phi(w, s, p) d\tilde{G}(w) - c(p) \geq \int_0^{\bar{w}} \Phi(w, \tilde{s}, \tilde{p}) d\tilde{G}(w) - c(\tilde{p}) \quad (14)$$

which proves the proposition.  $\square$

### 3.2. Discriminatory non-monetary sanctions

The optimal structure of the non-monetary sanction under the assumption that the sanction can vary with wealth has been analyzed by Polinsky (2006) and recently by D'Antoni et al. (2022).

The net cost of violations and sanctions as a function of the sanctioning strategy is still given by (9), but now  $s$  can be chosen to vary across individuals with different wealth  $w$ . This is to say that, for given  $p$ , the optimal sanctions  $f^*(w, p)$  and  $s^*(w, p)$  minimize the expression (9), while the optimal probability  $p$  takes into account the distribution of wealth, i.e., it solves

$$\min_p \int_0^{\bar{w}} \Psi(f^*(w, p), s^*(w, p), p) dG(w) + c(p). \quad (15)$$

In order to find the optimal level of the nonmonetary sanction  $s$  at each  $w$ , Polinsky (2006) assumes the problem admits an internal minimum, obtained by solving the first order condition:<sup>7</sup>

$$\Psi_s = -p[h + (1 + \gamma)s - b]r(b) + p(1 + \gamma)[1 - R(b)] = 0 \quad (16)$$

with  $b = p(f + s)$ .

The optimal sanctioning policy is characterized as follows (Polinsky, 2006, Proposition 3, p. 831). At levels of wealth where  $s^*(w, p) > 0$ :

- the optimal fine equals the offender's wealth, i.e.,  $f^*(w, p) = w$ ;
- the optimal imprisonment term decreases with wealth, i.e.,  $\partial s^*/\partial w < 0$ ;
- moreover, the optimal combined monetary and non-monetary sanction, i.e.,  $f^*(w, p) + s^*(w, p)$ , also decreases with wealth.<sup>8</sup>

Bearing in mind these conclusions, we can state the following:

**Proposition 3.** *Assume the social planner applies an optimal sanctioning policy characterized by a common probability  $p$ , a fine and a discriminatory nonmonetary sanction. Then, an equalizing transfer among individuals subject to a positive nonmonetary sanction determines an increase in the social costs of crime and law enforcement.*

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<sup>7</sup>D'Antoni et al. (2022) question the generality of this case, as there are quite common circumstances where the optimization problem does not admit an internal solution and the optimal nonmonetary sanction is either zero or maximal (where the maximum is defined exogenously). As pointed out by D'Antoni et al. (2023), for an internal solution to be a maximum, some additional assumptions about the shape of the distribution  $r$  are required, to make sure that the hazard rate is decreasing.

<sup>8</sup>A proof of this result is provided in the Appendix, section A.1.

*Proof.* Consider  $\Psi$  as defined in (9). We calculate:

$$\Psi_f = -p[h + (1 + \gamma)s - b]r(p(f + s)) < 0. \quad (17)$$

From (16) we have:

$$\Psi_f = -p(1 + \gamma)[1 - R(p(f + s))] \quad (18)$$

where the right hand side is always negative and it is increasing in the total sanction  $f + s$ .

With  $s > 0$  it is  $f^*(w, p) = w$ . Define  $\psi(w, p) = \Psi(w, s^*(w, p), p)$ . The envelope theorem implies that  $\psi_w = \Psi_f$ . Since, as shown by Polinsky (2006), the optimal total sanction  $w + s^*$  is decreasing in  $w$ , from (18) follows that, when the optimal sanction  $s^*$  is positive,  $\psi$  is decreasing and concave in  $w$ .<sup>9</sup>

The fact that  $\psi$  is concave whenever  $s > 0$  implies that a wealth equalizing redistribution among individuals subject to positive nonmonetary sanctions determines an increase in social cost.  $\square$

In this case, the conclusion we reach differs from propositions 1 and 2. The reason is that, while in the previous cases a redistribution of wealth to the poor increased deterrence on individuals with a lower benefit from violation, the fact that now the total sanction is higher for the poor implies the opposite. Namely, as long as they are subject to the nonmonetary sanctions, individuals who violate will have, on average, lower benefit from violation than poorer individuals.

Proposition 3 covers the case of individuals who are subject to a nonmonetary sanction. As illustrated in the example, some individuals, typically the richest, may be optimally subject only to a monetary sanction; a wealth redistribution among these individuals reduces the social cost. Because the effects of redistribution on social cost has different signs at different levels of wealth, we are not able to draw general conclusions on the effect of a reduction of inequality involving a redistribution across both groups of individuals.

Remark: the shape of  $r$  (Assumption 1) plays no role in this case!

#### 4. Discriminatory Enforcement

The analysis thus far assumed that enforcement was common to all offenders. We now show that, even when the sanction is only monetary, Proposition 1 does not carry over to the case where the social planner can discriminate among offenders by adjusting the level of enforcement to offender's wealth.

<sup>9</sup>An alternative proof of the concavity of  $\psi$  with respect to  $w$  is provided in the Appendix (section A.2), where we also show that Proposition 3 applies to the family of the Pareto distribution function with minimum parameter  $k$  and a shape parameter  $\alpha$ , satisfying the condition  $0 < \alpha < 1 + 1/\gamma$ .

When enforcement (represented by the probability  $p$ ) can be adjusted to offender's wealth, both  $f$  and  $p$  are optimally set to minimize

$$\int_{pf}^{\infty} (h - b)r(b)db + c(p). \quad (19)$$

under the usual constraint  $f \leq w$ .

When enforcement can be adjusted to wealth, there is no reason not to set the fine to its maximum level. This is so, because the fear of overdeterrence of individuals with high wealth can be avoided by selectively reducing  $p$ , thereby also saving enforcement costs. Hence, the optimal fine is  $f^*(w) = w$  for everyone.

The optimal enforcement probability  $p^*(w)$  for an individual with wealth  $w$  minimizes (19), hence it solves the first order condition:

$$-w(h - pw)r(pw) + c'(p) = 0. \quad (20)$$

As demonstrated by [Garoupa \(2001\)](#), the expected fine  $p^*(w)w$  is increasing in  $w$ ; however,  $p^*(w)$  may be either increasing or decreasing in  $w$ , so that enforcement and punishment may be either complements or substitutes. In particular,  $p$  and  $w$  will be complements ( $dp^*/dw > 0$ ) for low level of  $w$  and substitutes ( $dp^*/dw < 0$ ) for high level of  $w$ .

This affects the conclusion on the effect of a reduction in wealth inequality on the social cost, as stated in the following:

**Proposition 4.** *Assume the social planner applies an optimal sanctioning policy characterized by a common probability and a fine that can differ across individual depending on their wealth. Holding Assumption 1, an equalizing transfer may determine either an increase or a decrease of social costs. Namely, there will be an increase if the transfer affects only lower wealth individuals, and a decrease if it affects only higher wealth individuals.*

*Proof.* Let  $\psi(w) \equiv \Psi(w, p^*(w))$  represent the value of the social cost (19) at wealth  $w$  when  $f$  and  $p$  are chosen optimally. When the distribution of wealth is  $G$ , the social cost is  $\int_0^{\bar{w}} \psi(w)dG(w)$ , hence the effect of a change in distribution can be analysed looking at the shape of  $\psi$ . We have:

$$\psi' = -p(h - pw)r(pw) < 0 \quad (21)$$

$$\psi'' = p \frac{d(p^*w)}{dw} [r(pw) - (h - pw)r'(pw)] - \frac{dp^*}{dw} \cdot (h - pw)r(pw). \quad (22)$$

We see that, in the expression for  $\psi''$ , the first term is positive under Assumption 1, but

the second term is negative when  $dp^*/dw > 0$ , i.e. when  $p$  and  $w$  are complements; moreover, the second term may be large when  $w$  is small. This means that  $\psi$  can be concave at low levels and convex at high levels of wealth. Therefore, the argument used to prove Proposition 1 does not apply in this case when the equalizing transfers affect the whole distribution. In fact, when equalizing transfers take place within the interval where the social costs are concave (i.e., at low levels of wealth), a more equal distribution will increase social costs.  $\square$

To illustrate with an example, let us consider the simple case of uniform density  $r = 1$  over  $[0, 1]$ .<sup>10</sup> With  $h = .8$  and  $c(p) = p^2$  we have

$$p(w)w = \frac{0.8w}{w^2 + 2} \quad \Psi(w) = \frac{30 - w^2}{50(w^2 + 2)} \quad (23)$$

with  $\psi(w) \equiv \Psi(w, p^*(w))$  concave up to the point  $w = \sqrt{2/3}$ , and convex thereafter.

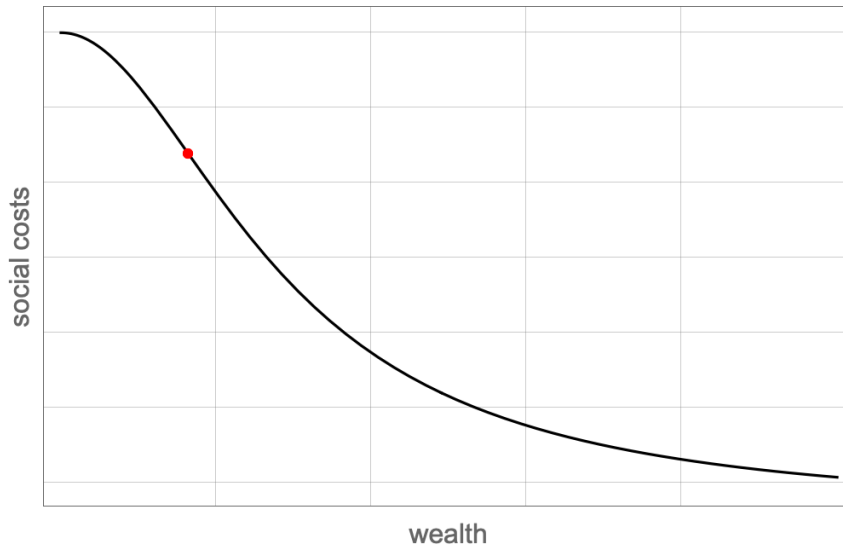


Figure 1:  $\psi(w)$  when  $h = 0.8$ ,  $c(p) = p^2$  and  $b$  is uniformly distributed on  $[0, 1]$

In this case, a reduction in inequality may have either a positive or negative impact on social cost, depending on which part of the wealth distribution is affected. In particular, an equalizing redistribution increases the social cost of law enforcement when the redistribution involves only low wealth individuals, so that  $\Psi(w)$  is concave at points where  $G$  and  $\tilde{G}$  differ.<sup>11</sup>

<sup>10</sup>Recall that a uniform density satisfies Assumption 1, so Proposition 1 applies.

<sup>11</sup>Specific numeric examples can be provided by the authors upon requests.

## 5. Discussion: cost of nonmonetary sanction and wealth

A number of paper discussing the relation between crime and inequality or poverty have considered that the individual cost of nonmonetary sanctions may depend on income or wealth (Polinsky and Shavell, 1984; Garoupa and Mungan, 2019). The idea is that the time of wealthier individuals may be more valuable, so their opportunity cost of their time spent in prison, or in other time consuming activities imposed as a sanction, is higher.

However, the assumption that the individual cost of  $s$  varies with wealth must be better qualified and may be questioned in our context.

If the source of the higher value of time is the circumstance that time is complementary to consumption, so a wealthier individual gets higher utility from his/her time when not in prison, it may be objected that, in our context, nonmonetary sanction is imposed only when individual wealth is zeroed out by the monetary sanction. This is to say that all individuals subject to a nonmonetary sanction will have the same, i.e. zero, wealth. Therefore, as long as their time value depends on available wealth at the time of the sanction, the marginal value of time should be the same for all.

A more sophisticated argument is that higher wealth reflects a higher level of human capital. An individual with high wealth has also a higher productivity, hence imprisonment or other restrictions to his/her ability to work imply a higher individual cost. In this case, the cost is not affected by a monetary sanction that takes all his/her wealth.

However, even in this case the argument does not appear entirely sound. First, if we take a longer term perspective, where future earnings are taken into account, a more precise definition of wealth should include also human capital. In fact, in many cases, when an individual is judgement proof, the legal system stipulates some form of installment payment to be paid off in the future, which corresponds to a claim on human capital. This is to say that the individual may not benefit of his/her higher human capital. Moreover, in addition to a higher individual cost of incarceration, removing a higher-productivity individual from work could entail a higher social cost. This means that it is not clear that the ratio of individual to social cost of imprisonment is affected in one direction or the other.<sup>12</sup>

Despite these caveats, we have considered how the assumption that imprisonment is more costly at higher levels of wealth affects our conclusions. We found that the results of section 3, namely propositions 2 and 4, remain the same under this assumption. A formal analysis is provided in the [Appendix](#).

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<sup>12</sup>If we interpret  $s(w)$  as the monetary cost of imprisonment to an individual with wealth  $w$ , assuming such cost is increasing in  $w$  is equivalent to assuming that the social cost  $\gamma$  is decreasing in  $w$  (imposing to a richer individual a given cost requires a shorter term in prison, hence the social cost  $\gamma$  is lower).

## 6. Conclusions

We have shown that in the case of monetary sanctions, even when benefits from violations are independent of wealth, an equalizing wealth redistribution decreases the social cost of law enforcement.

On the other hand, when both monetary and nonmonetary sanctions are used, the conclusion differs depending on the existence of constraints in the use of nonmonetary sanctions. Namely, when the enforcer cannot discriminate among individuals based on their wealth, the result of monetary sanctions carries on to the case of joint monetary and nonmonetary sanctions.

Notably, the result has been obtained under the assumption that the social planner applies the same enforcement policy to all offenders, regardless of their wealth. When the ability to discriminate according to wealth extends also to the enforcement policy, the opposite conclusion, namely a positive association between social cost and distributive equity, may result in some cases.

Our conclusions can be interpreted as establishing a complementarity between equity in criminal justice and distributional equity, in the sense that (especially in poorer societies where the level of wealth is low) nondiscrimination in enforcement and in the imposition of nonmonetary sanctions are necessary condition for a more equal distribution of wealth to reduce the cost of law enforcement.

Of course, our analysis is not suggesting that the effect on the social cost of enforcement is the only, or the most relevant, positive effect of redistribution. Our more limited claim is that, on top of other welfare effects and under some condition of “criminal equity”, redistribution will have some benefits on the cost of enforcement. Noteworthy, such effect is independent of the possible direct effect of wealth on individual benefits from violations.

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## APPENDIX

### A.1. When nonmonetary sanctions are discriminatory, the total sanction is decreasing with wealth

This proof follows the one provided in Proposition 3 by Polinsky (2006). Consider the expression (9). Let the optimal sanctions at wealth  $w$  be  $s(w) > 0$ , so that  $f(w) = w$  and the first order condition (16) is satisfied, that is:

$$-p[h + (1 + \gamma)s(w) - b(w)]r(b(w)) + p(1 + \gamma)[1 - R(b(w))] = 0 \quad (24)$$

with  $b(w) = p(s(w) + w)$ . Consider now a lower level of wealth  $w' < w$ . At wealth  $w'$ , we set the nonmonetary sanction at the level  $s'$  such that the total sanction is  $s' + w' = s(w) + w = b(w)$ , so that  $s' = s(w) + (w - w') > s(w)$ . The expression in (24) is:

$$\begin{aligned} & -p[h + (1 + \gamma)s' - b(w)]r(b(w)) + p(1 + \gamma)[1 - R(b(w))] < \\ & -p[h + (1 + \gamma)s(w) - b(w)]r(b(w)) + p(1 + \gamma)[1 - R(b(w))] = 0. \end{aligned} \quad (25)$$

Therefore, at wealth  $w'$ , an expected total sanction equal to  $b(w)$  is less than optimal, as an increase of  $s$  above  $s'$  reduces the social cost. This implies that  $s(w') + w' > s(w) + w$ , which proves the conclusion.

### A.2. Concavity of $\psi$ in Proposition 3

We provide a further prove that  $\Psi$  is convex when the nonmonetary sanction is discriminatory (as stated in the proof of Proposition 3).

Consider  $\Psi(f, s)$  and let  $f^*(w) = w$  and  $s^*(w)$  be the optimal sanctions (where we have omitted the argument  $p$  for notational simplicity). We want to check that  $\psi(w) \equiv \Psi(w, s^*(w))$  is decreasing and concave in  $w$ .

From the envelope theorem we have that:

$$\psi' = \Psi_f < 0 \quad \psi'' = \Psi_{ff} + \frac{ds^*}{dw} \Psi_{fs}. \quad (26)$$

If  $s^*(w)$  minimizes  $\Psi$ , then  $\Psi_s = 0$  and  $\Psi_{ss} > 0$ . From the first order condition follows (implicit derivative):

$$\frac{ds^*}{dw} = -\Psi_{sf}/\Psi_{ss} \quad (27)$$

so that, substituting in the expression of  $\psi''$  we have:

$$\psi'' = \frac{\Psi_{ss}\Psi_{ff} - \Psi_{fs}^2}{\Psi_{ss}}. \quad (28)$$

We calculate from (9):

$$\Psi_{ff} = p^2[r(b) - r'(b)(h - pf + \gamma ps)] \quad (29)$$

$$\Psi_{ss} = -p^2[(2\gamma + 1)r(b) + r'(b)(h - pf + \gamma ps)] \quad (30)$$

$$\Psi_{fs} = p^2[\gamma r(b) - r'(b)(h - pf + \gamma ps)], \quad (31)$$

with  $b = p(f + s)$ . Then

$$\Psi_{ss}\Psi_{ff} - \Psi_{fs}^2 = -p^4(1 + \gamma)^2 r(b)^2 < 0. \quad (32)$$

This implies that  $\psi'' < 0$ , so that  $\psi$  is concave.

#### A.2.1. An example with the Pareto distribution

Consider that the benefits from violations are distributed according to a Pareto distribution<sup>13</sup> with minimum parameter  $k$  and shape parameter  $\alpha$ .

The density function

$$r(b) = \begin{cases} \alpha b^{-(1+\alpha)} k^\alpha & b \geq k \\ 0 & b < k. \end{cases} \quad (33)$$

We have

$$\Psi(f, s) = \int_{p(f+s)}^{\infty} (h + ps(1 + \gamma) - b) \alpha b^{-(1+\alpha)} k^\alpha db + c(p) \quad (34)$$

The first order condition with respect to  $s$  for an interior solution is:

$$pk^\alpha (p(f + s))^{-(1+\alpha)} (fp(\alpha + \gamma + 1) - \alpha h + ps(1 + (1 - \alpha)\gamma)) = 0 \quad (35)$$

Substituting  $f = w$  and solving for  $s$  we have that

$$s^*(w) = \frac{p(1 + \alpha + \gamma)w - \alpha h}{p(1 + (1 - \alpha)\gamma)} \quad (36)$$

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<sup>13</sup>The Pareto distribution is suitable to illustrate the case of an optimal nonmonetary sanction because it satisfies the condition on the hazard rate stated by [D'Antoni et al. \(2022, 2023\)](#).

The derivative of  $s^*(w)$  is

$$\frac{ds^*}{dw} = -\frac{1 + \alpha + \gamma}{1 + (1 - \alpha)\gamma} \quad (37)$$

which is negative as long as:

$$0 < \alpha < \frac{1 + \gamma}{\gamma} \quad (38)$$

When the derivative is negative, it is less than  $-1$ , so that:

$$\frac{d(s^*(w) + w)}{dw} < 0, \quad (39)$$

implying that deterrence decreases with  $w$ .

Plugging  $s^*(w)$  in the social costs function and differentiating with respect to  $w$  we obtain:

$$\psi'(w) = (\gamma + 1)p(-k^\alpha) \left( \alpha \frac{(\gamma + 1)wp - h}{(\alpha - 1)\gamma - 1} \right)^{-\alpha} < 0 \quad (40)$$

and

$$\psi''(w) = \frac{\alpha(\gamma + 1)^2 p^2 k^\alpha}{(\gamma + 1)wp - h} \left( \alpha \frac{(\gamma + 1)wp - h}{(\alpha - 1)\gamma - 1} \right)^{-\alpha} < 0 \quad (41)$$

which confirms concavity.

The conclusion on concavity can be illustrated through a numerical example, assuming the benefits are distributed according to a Pareto distribution with minimum value 1 and a shift parameter 2. The expected value of these benefits is 2. Let us also assume that  $h = 5$  and  $\gamma = 0.2$ , so the condition is indeed satisfied. The optimal non-monetary sanction is given by  $s^* = 25 - 4w$ , which decreases with  $w$ . The optimal social costs of crime and law enforcement (disregarding the costs of enforcement) are given by:

These costs are illustrated in figure 2. As it is evident, the social costs are concave for  $1 < w < 6.25$ . For  $w > 6.25$ , the optimal imprisonment term is 0, and we are back to 1, where the social costs are convex.

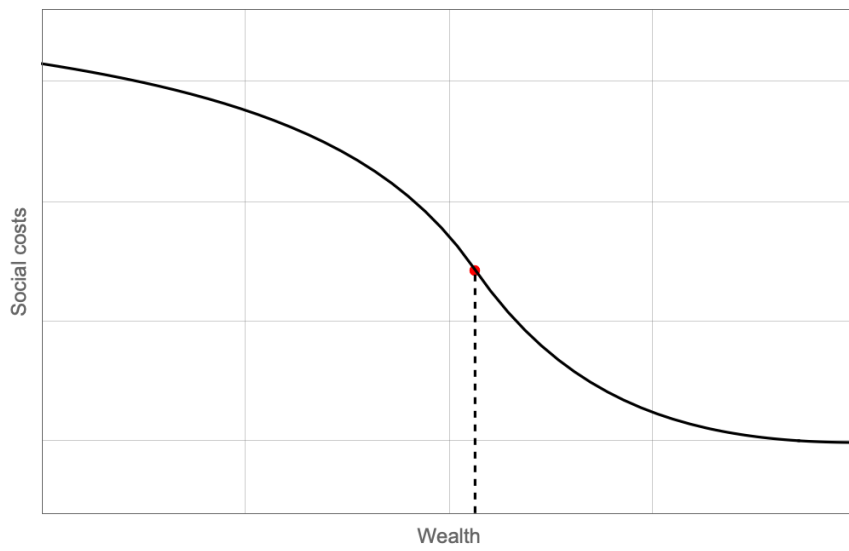


Figure 2:  $\phi(w, p)$  when  $h = 5$ ,  $p = 0.5$ ,  $\gamma = 0.2$  and  $b \sim \mathcal{P}(1, 2)$