Firm Dynamics after COVID-19: A Structural Model

Guzmán González-Torres*

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Abstract

I study the effects of the COVID-19 pandemic on firm dynamics and the real economy, and evaluate the efficacy of various policies designed to lower firms' liquidity needs. I propose a firm dynamics model with investment in physical capital, in which firms seek out external liquidity in order to finance ongoing operations. In response to an unexpected fall in aggregate demand as well as temporary workforce reductions due to stay at home orders, firm exit increases and entry decreases. The pandemic has lasting effects on the stock of firms and aggregate output, but only transitory effects on firm selection and aggregate growth. I find that policies which directly reduce fixed costs can have a significant effect on the evolution of the real economy in the long run by curbing firm exit in the immediate aftermath of an aggregate shock. However, the policies need to be in place several periods in the worst hit sectors of the economy in order to be effective because firms expect aggregate demand to only recover slowly. Measures attempting to lower firm total payrolls on the other hand are only effective in sectors heavily affected by stay at home orders.

1 Introduction[1]

The COVID-19 pandemic has created serious liquidity problems for firms by limiting their ability to produce revenues for a considerable time. On the one hand, stay at home

*Bank of Italy, Directorate General for Economics, Statistics and Research, Law and Economics Division. Please reach me at guzman.gonzalez-torres@bancaditalia.it. The views expressed in this paper are my own and do not necessarily reflect those of the Bank of Italy.

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orders imposed due to health concerns curbed production in sectors not deemed essential by impeding workers from reaching their workplaces in so far as their tasks weren't doable remotely. On the other hand, the recession caused by the pandemic has negatively affected sales for the foreseeable future. In the hope of limiting a looming wave of firm destruction, governments around the world have responded by implementing policies to decrease firm liquidity needs, such as enabling firms to furlough workers without having to compensate them, or letting firms postpone long-run debt payments.

Although evidence points to the overall effectiveness of the set of measures applied so far, several open questions remain regarding the relative efficacy of the different policies, the effects on aggregate productivity through selection they might have, as well as what the best policy mix might be looking forward, as fiscal space diminishes with the endurance of some public health measures. Structural models in which short-run liquidity plays a central role in determining firm entry, investment, and exit decisions, can help policy makers evaluate the aggregate consequences of firm dynamics reacting to the initial shock and the aforementioned emergency measures, as well as design effective phaseout schemes as the pandemic recedes. In this paper I propose a model of an industry with entry and exit, in which firms endogenously seek out external liquidity in order to finance their current operations.

The model economy is populated by potential entrants and price-taking operating firms. Firms are heterogeneous in their permanent productivity, their transitory productivity, and their capital stock. Firms that decide to operate in the final good market need to obtain external liquidity in order to finance their inputs. Given firms face a fixed production cost, only those productive enough to cover it are able to secure enough funds to operate. They may however decide to endogenously liquidate their capital stocks and exit the market at any time. Firms that do produce, must decide whether to employ their earnings to invest in capital for the following period, or to distribute them to firm owners. The latter can choose to inject additional funds in their firms, which are also used to invest in capital. Finally, potential entrants can choose to invest in capital at the end of a period in order to attempt entering the market in the following period. All firms make their investment decisions before knowing their productivity for the following period.

I assume that all prices, i.e. the final good price and all input prices, are determined outside the model.[2] There is no aggregate uncertainty in the model: even though individual firms evolve stochastically across time, they expect the industry as a whole to

[2]The quantitative exercise I conduct can thus be thought of as representing a single sector small enough to not have an effect on input prices. Although endogenizing both input and output prices does not represent a conceptual challenge, it would add substantial computational complexity to the solution of the model. I leave this extension to future research.

remain in an ergodic state indefinitely. Starting from such an equilibrium, I introduce the economic effects of the COVID-19 pandemic by simulating how the economy evolves as a whole if two of its characteristics unexpectedly change. First, the price of the final good, which I use as a proxy of aggregate demand conditions, falls as the pandemic hits the economy and later follows a deterministic path back to its original level during several periods. Second, the pandemic temporarily reduces the amount of labor initially hired by firms that they can effectively utilize in production. I assume firms have rational expectations: although they do not initially anticipate a change in either variable, they have perfect foresight regarding their trajectories after the initial shock.

I calibrate the initial ergodic equilibrium to a set of moments representative of the dynamics and size distribution of Italian firms, as computed from microdata from the Italian firm registry. I also use two pieces of information from sectoral data to inform my calibration. First, I back out the portion of employees who were unable to work during the 2020 lock downs. I compute it as the percentage of employees in non-essential sectors whose tasks can't be completed remotely and use it as a direct input to discipline the labor utilization constraints faced by firms. Second, I estimate the fall in revenues, which I use to calibrate the unexpected price fall in my quantitative exercise. I use this calibration to explore the effects of implementing two types of policies designed to alleviate firms' liquidity shortages: government subsidized furloughs (CIG) and long-run debt moratoria (MOR).

In the absence of government intervention, the COVID-19 shock produces a spike in firm exit on impact: both bankruptcy and voluntary liquidation increase, though the latter is significantly more affected. Given the entry rate falls simultaneously, the number of firms in the economy falls. Nonetheless, the firms that leave the market are the least productive. Aggregate employment, investment, and output take a significant hit on impact, as do aggregate borrowing and profits. Aggregate dividend payouts increase, as the value of investment falls due to the shock. In the long run, the shocks to labor utilization and the price of the final good only display temporary effects on exit, entry, and growth rates in this economy. However, the slow recovery of the stock of firms leaves long lasting effects, as aggregate employment, capital, and output require significant time to return to their original levels.

In order to explore whether the effectiveness of CIG and MOR depend on the intensity of each shock, I simulate the economy under four scenarios representing sectors of the economy diversely affected by falls in demand and stay at home orders respectively. MOR effectively lowers firms' liquidity needs under all scenarios, by letting them postpone certain pre-existing financial obligations. This helps lower bankruptcy rates, although at the cost of partially undoing the positive selection at exit the COVID-19 shock generates.

CIG on the other hand only proves to be effective in scenarios in which stay at home orders prevent firms from utilizing a significant portion of their workforce, while eroding less of the positive selection at exit.

All shock combinations elicit qualitatively identical responses in the absence of policy interventions, yet the fall in the price of the final good dominates the introduction of the labor utilization constraint, as the slow return to its original level significantly impacts firms' continuation values. Consequently, neither policy intervention is able to reverse the incentive to voluntarily liquidate that firms face under a severe fall in the price of the final good, even when they are effective at preventing a wave of bankruptcies. This result suggests that although the choice of policies implemented so far might have been appropriate, governments might need to slow down the pace at which they retract them in the sectors hit worst.

The results of my quantitative exercise directly talk to the rapidly expanding evidence using different approaches, that points at the severity of the pandemic for firm dynamics and that the various government relief schemes have in fact been effective. Using balance sheet data for Italian firms and tractable accounting frameworks, Schivardi and Romano (2020) and Orlando and Rodano (2020) estimate that the aforementioned government schemes might have kept 100,000-200,000 firms afloat in Italy in 2020. Gourinchas et al. (2020) propose a structural model at the firm level to provide a range of estimated bankruptcy rate spikes at impact of the COVID-19 pandemic in line with my findings.[3] Elenev et al. (2020) use a macroeconomic model of financial intermediation with both financial and non-financial corporate default to study the effectiveness of various policies implemented in the US, finding that different programs vary not only in the amount of non-financial firms they manage save from bankruptcy, but also in the type of firms they reach.

The main structure of my model is based on the seminal works of Jovanovic (1982) and Hopenhayn (1992), which have become the canonical models describing the mechanisms behind firm selection in and out of industries. A myriad of different versions of these, such as Clementi and Palazzo (2016) or Carvalho and Grassi (2019), tackle important policy issues regarding the aggregate effects of firm dynamics, including Cooley and Quadrini (2001) and Chatterjee et al. (2007), which introduce corporate finance as an important element in this literature. The novelty of the approach I take in this paper is that of introducing involuntary default through endogenously determined liquidity needs. This paper is also directly related to the extensive literature that studies the macroeconomic effects of financial frictions through entrepreneurship, such as Cagetti and De Nardi (2006),

[3] I also refer to them for an excellent overview of the extensive quantitative literature on the effects of COVID-19.

Midrigan and Xu (2014), Buera and Moll (2015), or Khan et al. (2016). My approach differs from these models in two general ways: firstly, by fully endogenizing firm entry and exit, and secondly by focusing on liquidity needs, rather than financial frictions arising through collateralized lending.

The paper is organized as follows: Section 2 describes the model economy; Section 3 illustrates the calibration of the model to the Italian economy; Section 4 describes the quantitative exercises I run to illustrate the main mechanisms of the model; and Section 5 concludes.

2 Model

The economy is populated by a continuum of infinitely-lived firms consisting of a mass $M_{0,t}$ of potential entrants, and a mass $M_{i,t}$ of operating firms.[4] Time is discrete and a period in the model represents a natural calendar year. All firms are price takers and all prices are determined exogenously. There is no aggregate uncertainty in the economy, but firms are heterogeneous along three dimensions: their permanent productivity level θ, their transitory productivity shock ε, and their installed stock of capital K.

Given the functional and parametric assumptions of the model outlined below, the entire dynamic problem firms face is normalizable by a firm's permanent productivity level.[5] In what follows, lower-case variables represent quantities normalized by permanent productivity: $x = \frac{X}{\theta}$.[6] Note also that I present the model in recursive form absent any time subscripts up to the equilibrium definition, so that a variable x' represents the variable x one period ahead.

Firms

The sequence of events firms face in a given period is summarized in Figure 1. At the beginning of a period, firms need to decide whether to enter the final good market. Producing implies paying for inputs upfront; lacking liquidity, firms that decide to do so need to obtain access to short-term credit, which is only available to them if their current

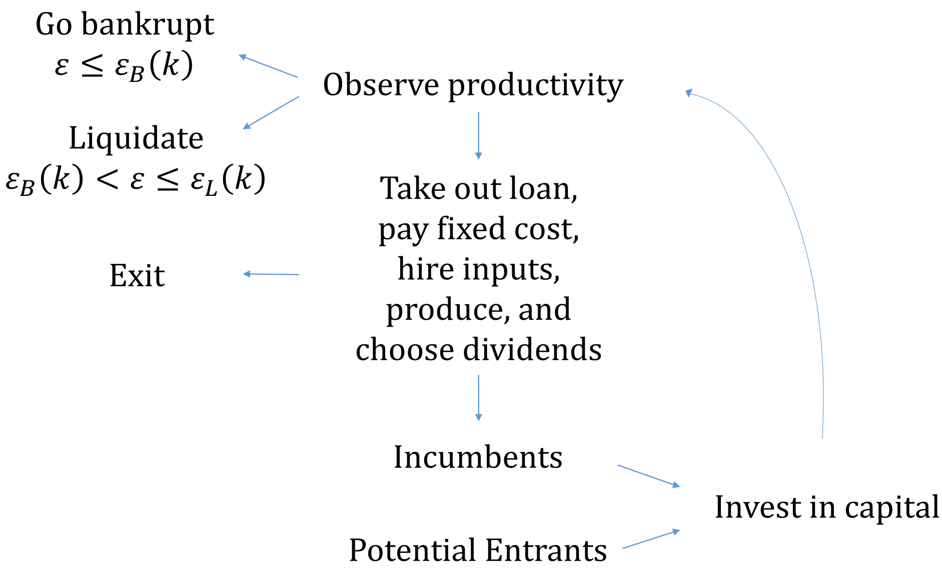
[4]The mass of firms and mass of potential entrants can vary along the transition between ergodic steady states, as can prices and firm choices. However, firms only need their individual state variables (described below) and the current and one period ahead expected price in order to solve their optimization problems. I thus drop all time subscripts up to the equilibrium definition.

[5]I use the production function proposed by Midrigan and Xu (2014). As fully discussed there, incorporating permanent productivity is crucial to correctly replicating the distribution of firm revenue in the data. See Appendix B.1 for details of the model specified in levels.

[6]Note that in an abuse of terminology, I refer below to normalized quantities as the quantities themselves; e.g. I refer to k as the capital stock instead of the normalized capital stock.

productivity and capital stock warrant sufficient revenues at the end of the period to repay borrowing costs. Firms that do produce, must decide whether to continue operating an additional period. Those that choose to do so, can employ their earnings to invest in capital for the following period and distribute dividends. Firm owners can choose to inject additional funds in the firm, which are also used to invest in capital. Finally, potential entrants can invest in capital at the end of a period in order to attempt entering the market in the following period. All firms make their investment decisions before knowing their productivity for the following period.

Figure 1: Timing of the Model



In the following first two paragraphs I describe the choices regarding production and investment faced by firms that operate in the final good market. The ensuing two paragraphs lay out firm entry and exit decisions.

Input choices At the beginning of a period, firms observe their individual capital stocks k and transitory productivity shocks ε. Firms can pay a fixed cost c_f to produce and sell a final consumption good at the competitive price p. They use labor $l(k, \varepsilon)$ at price w and intermediates $m(k, \varepsilon)$ at price p_m as variable inputs. Capital, which is the only fixed physical input, depreciates at rate δ when used in production. Given that firms do not hold any assets other than installed capital, and that production costs need to be paid upfront, they are obliged to borrow $c_f + wl(k, \varepsilon) + p_m m(k, \varepsilon)$ at a unit financial cost r_s

in order to participate in the market.

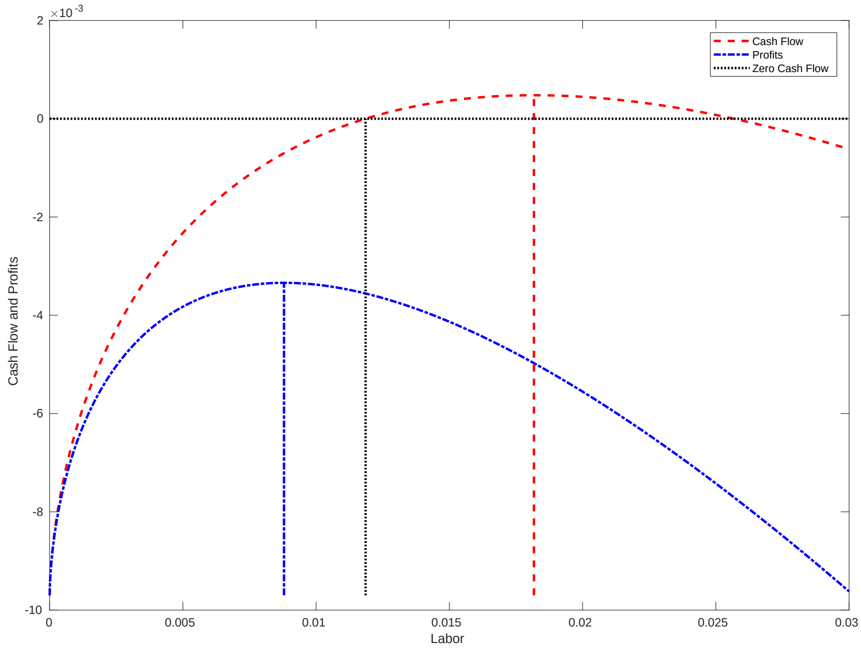
$$CF\left(l,m,k,\varepsilon\right)=p\varepsilon^{1-\nu}\left(l\left(k,\varepsilon\right)^{\alpha}k^{\beta}m\left(k,\varepsilon\right)^{\gamma}\right)^{\nu}-\left(wl\left(k,\varepsilon\right)+p_{m}m\left(k,\varepsilon\right)+c_{f}\right)\left(1+r_{s}\right) \quad (1)$$

I assume firms use a span of control technology, where $\nu\in[0,1]$ determines the managerial and physical input elasticity of output, and $\{\alpha,\beta,\gamma\}$ respectively determine the labor, capital, and intermediate input elasticities of output. Furthermore, I assume that $\alpha+\beta+\gamma=1$, which implies decreasing returns to scale to physical inputs. Equation 1 shows cash flow before tax for firms that obtain outside liquidity, which is equal to value added net of fixed costs and financing costs. I assume that firms need to repay their short term loans at the end of the period, which they are only able to whenever $CF\left(l,m,k,\varepsilon\right)>0$. In other words, they can only produce if they are productive enough to cover their financial expenses with their revenues.

$$\pi\left(l,m,k,\varepsilon\right)=\begin{cases}CF\left(l,m,k,\varepsilon\right)-\tau_{w}wl\left(k,\varepsilon\right) & ,\, CF\left(.\right)-\tau_{w}wl\left(k,\varepsilon\right)<\delta k\\ \left(1-\tau_{c}\right)\left(CF\left(l,m,k,\varepsilon\right)-\tau_{w}wl\left(k,\varepsilon\right)\right)+\tau_{c}\delta k & ,\, CF\left(.\right)-\tau_{w}wl\left(k,\varepsilon\right)\geq\delta k\end{cases} \quad (2)$$

Firms owe payroll tax at the proportional rate τ_w, which they pay out of their cash flow. Additionally, if their cash flow after payroll tax and capital depreciation is positive, they owe corporate income tax at rate τ_c. Equation 2 summarizes firm profits after financing costs, depreciation, and tax.

Figure 2: Cash Flow and Profits as a Function of Labor



Firms choose labor and intermediate inputs to maximize profits, subject to producing enough revenues to cover the cost of outside liquidity. As illustrated in Appendix B.2, maximizing profits is equivalent to statically maximizing cash flow minus payroll tax with respect to $\{l, m\}$. Note that payroll tax thus creates a wedge on labor, so that firms choose to produce at a lower scale $l_\pi(k,\varepsilon) \equiv \arg\max_l \{\max_m \pi(l,m,k,\varepsilon)\}$ than the cash flow maximizing amount of labor $l_{CF}(k,\varepsilon) \equiv \arg\max_l \{\max_m CF(l,m,k,\varepsilon)\}$. As illustrated in Figure 2, some firms might not produce high enough cash flow at their profit-maximizing scale $l_\pi(k,\varepsilon)$ to cover the cost of obtaining outside liquidity and thus afford paying for their physical inputs. In such cases, a firm that chooses to produce maximizes its profits (minimizes its losses) by increasing its hiring up to the point at which it produces just enough revenues to cover the cost of external liquidity. I define the resulting profit function in Equation 3, where $CF_m(k,\varepsilon) \equiv \max_{l,m} CF(l,m,k,\varepsilon)$ represents the maximized cash flow function.

$$\pi_m(k,\varepsilon) \equiv \begin{cases} \max_{l,m} \pi(l,m,k,\varepsilon) & , CF_m(k,\varepsilon) \geq 0 \\ s.t.\, CF(l,m,k,\varepsilon) \geq 0 & \\ 0 & , otherwise \end{cases} \tag{3}$$

Investment Firms that decide to continue operating at the end of a period have to decide how much capital k' to invest in for the following period.[7] Equation 4 formally describes the trade-off firms face when choosing k', summarized in the value of investing $V_i(k, \varepsilon)$.

$$\begin{aligned} V_i(k, \varepsilon; p) = & \max_{k'} - (k' - (1-\delta) k) - c_a((1-\delta) k, k') + \frac{1}{1+r} E_{\varepsilon'|\varepsilon} V(k', \varepsilon'; p') \\ & s.t.\, k' \geq 0 \\ & \quad \ln \varepsilon' = (1-\rho) \mu + \rho \ln \varepsilon + \eta',\, \eta' \sim N\left(0, \sigma^2\right) \end{aligned} \tag{4}$$

On the one hand, investing in capital increases firm expected continuation value conditional on current productivity, $E_{\varepsilon'|\varepsilon} V(k', \varepsilon'; p')$, which firms discount at rate r. Firms do not know their productivity in the next period ε', but are aware it is drawn from a log-normal distribution with mean μ, dispersion σ^2, and persistence ρ. However, firms perceive no uncertainty regarding the final good price in the following period p'. Firm continuation value additionally depends on the entry and exit decisions described further below. On the other hand, firms face capital adjustment costs $c_a(k, k')$, which take the form illustrated in Equation 5, on top of their capital investment expenses $(k' - (1-\delta) k)$.

$$c_a(k, k') = c_0^a k + c_1^a \left(\frac{k' - k}{k}\right)^2 k \tag{5}$$

Potential entrants have to invest in a starting capital $k_e'(p')$. Their investment and adjustment costs are conditioned by the fact that they have no previous capital. Additionally, I assume that the entering productivity for any given potential entrant, unknown at the time of investing, is distributed following the unconditional population distribution of productivity shocks in the economy $G(\varepsilon')$. Equation 6 defines the investment problem of a potential entrant.

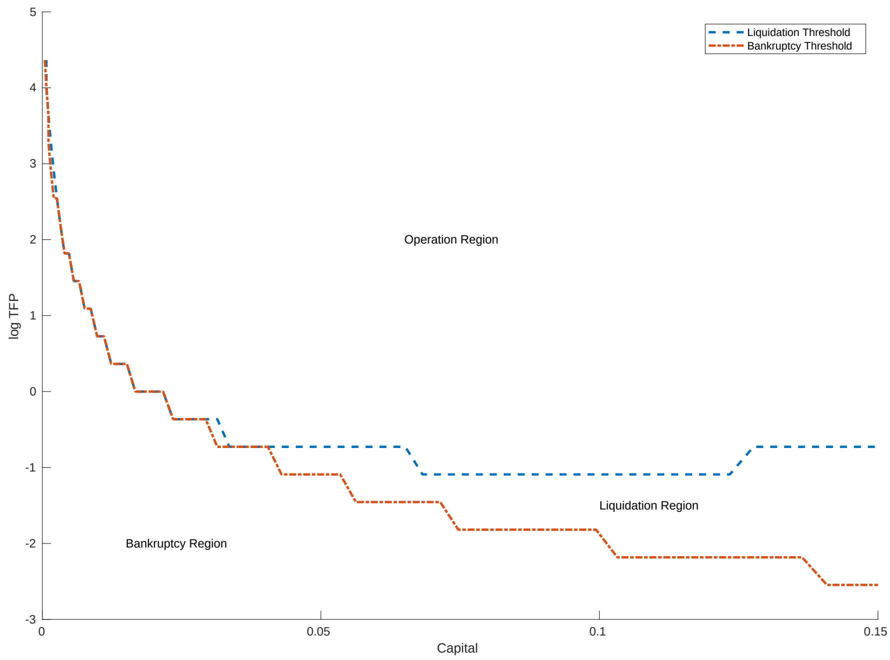
$$\begin{aligned} k_e(p') = & \arg\max_{k'} -k' - c_a(0, k') + \frac{1}{1+r} E_{\varepsilon'} V(k', \varepsilon'; p') \\ & s.t.\, k' \geq 0 \\ & \varepsilon' \sim G(\varepsilon') \end{aligned} \tag{6}$$

Note that investment is the only dynamic choice faced by firms in this economy. As such,

[7] I assume firms can only finance their capital stock through retained earnings and costless external equity, so that their investment decisions implicitly determine dividend payouts. In the absence of frictions that brake the Modigliani Miller theorem, assuming firms could finance capital investment with debt would create an indeterminacy in the choice of liabilities. My assumption thus does not affect the results of the model.

Equations 4 and 6 are the only parts of firms' individual problems in which the future evolution of prices enter. I therefore explicitly include the final good price as an argument of all value functions, while dropping it as an explicit argument in all other functions.

Figure 3: Numerical Approximation of the Bankruptcy and Voluntary Liquidation Thresholds



Obtaining liquidity There is no uncertainty regarding firms' capacity to honor their financial commitments, given that transitory productivity and the capital stock are known when they seek out external liquidity at the beginning of a period. Consequently, creditors solely lend funds to firms which produce sufficient revenues to repay their liquidity costs in full, i.e. firms with positive cash flow. As shown in Appendix B.2, the maximized cash flow function $CF_m(k, \varepsilon)$ is monotonically increasing and concave in both capital and transitory productivity. Consequently, I define the bankruptcy function $\xi_B(k, \varepsilon)$ as illustrated in Equation 7.

$$\xi_B(k, \varepsilon) = \begin{cases} 0 & , CF_m(k, \varepsilon) \geq 0 \\ 1 & , otherwise \end{cases} \tag{7}$$

Because cash flow is monotonically increasing in transitory productivity, there is a unique transitory productivity threshold $\varepsilon_B(k)$ for every capital stock level k such that firms with productivity below will not receive external liquidity, thus going bankrupt. Equation 8 defines the bankruptcy threshold, which is monotonically decreasing in both installed capital, as shown in Figure 3, and the final good price.

$$\varepsilon_B(k) = \left(\frac{1+r_s}{\nu p k^{\beta\nu}} \left(\frac{w}{\alpha}\right)^{\alpha\nu} \left(\frac{p_m}{\gamma}\right)^{\gamma\nu} \left(\frac{c_f}{(\nu^{-1}-\alpha-\gamma)}\right)^{1-\nu(\alpha+\gamma)} \right)^{\frac{1}{1-\nu}} \tag{8}$$

Firms that do not obtain financing liquidate their capital stock and exit the market bearing a proportional capital liquidation cost $c_l k$. The value of liquidating without entering the market in a given period $V_l(k)$ is composed of the sale value of installed capital minus the aforementioned liquidation cost, as illustrated in Equation 9.

$$V_l(k) = k - c_l k \tag{9}$$

Finally, note that firms above the bankruptcy threshold might endogenously decide to liquidate. Given the value of liquidation is constant with respect to productivity, that the decision to produce implies depreciating capital, and that profits and the option value of continuing are increasing in productivity, there is a threshold $\varepsilon_L(k) \geq \varepsilon_B(k)$ such that firms below it decide to liquidate without producing even in the face of positive cash flow.

$$\xi_L(k,\varepsilon) = \begin{cases} 0 & , \pi_m(k,\varepsilon) + \max\{V_i(k,\varepsilon;p), V_e(k)\} \geq V_l(k) \\ 1 & , otherwise \end{cases} \tag{10}$$

Equation 10 defines the voluntary liquidation function, which is also pictured in Figure 3. The option value of continuing is equal to the envelope of the value of investing and the the value of exiting after production $V_e(k)$, which I introduce in the following paragraph.

Continuation value of a firm Operating firms can choose to invest in capital in order to remain active until the next period, thus obtaining $V_i(k,\varepsilon)$, or exit immediately after receiving their payoffs from production. The value of exiting $V_e(k)$ amounts to the resources obtained from liquidating depreciated capital, as shown in Equation 11.

$$V_e(k) = (1-\delta)k - c_l(1-\delta)k \tag{11}$$

I summarize the exit decision through the exit function $\xi_E(k,\varepsilon)$, defined in Equation 12.

$$\xi_E(k,\varepsilon) = \begin{cases} 0 & , V_i(k,\varepsilon;p) \geq V_e(k) \\ 1 & , otherwise \end{cases} \tag{12}$$

Having characterized all exit and entry decisions firms face, I present the continuation value of a firm in Equation 13. The first term represents firm liquidation value in case of bankruptcy. The term inside braces shows the choice between voluntarily liquidating and participating in the market, for firms with access to external liquidity. Finally, the term within brackets constitutes the envelope of the value of exit and the value of investing in capital an additional period, which determines the firms that exit after production.

$$\begin{aligned} V(k,\varepsilon;p) =& \xi_B(k,\varepsilon)V_l(k) + (1-\xi_B(k,\varepsilon))\{\xi_L(k,\varepsilon)V_l(k) \\ &+ (1-\xi_L(k,\varepsilon))[\pi_m(k,\varepsilon) + \xi_E(k,\varepsilon)V_e(k) \\ &+ (1-\xi_E(k,\varepsilon))V_i(k,\varepsilon;p)]\} \end{aligned} \tag{13}$$

Note that potential entrants with given capital stocks k and productivity levels ε are indistinguishable from incumbents with the same characteristics.

Equilibrium

Given prices $\{p_t, w_t, p_{m,t}\}_{t=0}^{\infty}$ are set exogenously, I define a forward-looking equilibrium of the economy as a set of input choices $\{l(k,\varepsilon;p_t)\}_{t=0}^{\infty}$ and $\{m(k,\varepsilon;p_t)\}_{t=0}^{\infty}$ that solve Equation 2, investment policy functions $\{k'(k,\varepsilon;p^{t+1})\}_{t=0}^{\infty}$ for incumbents and $\{k'_e(p_{t+1})\}_{t=0}^{\infty}$ for potential entrants that satisfy Equations 4 and 6, the corresponding bankruptcy functions $\{\xi_B(k,\varepsilon;p_t)\}_{t=0}^{\infty}$ emerging from Equation 7, the liquidation $\{\xi_L(k,\varepsilon;p_t)\}_{t=0}^{\infty}$ and exit decisions $\{\xi_E(k,\varepsilon;p_t)\}_{t=0}^{\infty}$ defining firm continuation value in Equation 13, and the distribution of firms at the beginning of each period $\{\Phi_t(k,\varepsilon)\}_{t=0}^{\infty}$ that emerge from firm entry and exit decisions as determined by Equation 14, conditional on the mass of potential entrants in every period $\{M_{0,t}\}_{t=0}^{\infty}$.[8]

$$\begin{aligned} \Phi_t(k',\varepsilon') =& \mathbb{1}_{\{k'_{e,t}=k'\}} M_{0,t} \int dG(\varepsilon) \\ &+ \int\int (1-\xi_e(k,\varepsilon;p_t))(1-\xi_l(k,\varepsilon;p_t))(1-\xi_B(k,\varepsilon;p_t)) \\ &\mathbb{1}_{\{k'_t(k,\varepsilon)=k'\}} P(\varepsilon'|\varepsilon)\, d\Phi_t(k,\varepsilon) \end{aligned} \tag{14}$$

The previous equilibrium definition applies without regards to whether prices, the mass of potential entrants, and the distribution of firms are constant across periods or not. I define an ergodic steady state equilibrium as a special case of the aforementioned equilibrium, in which all prices are constant and all time subscripts can be dropped, as all equilibrium

[8] In the baseline version of the model, I set the mass of potential entrants to a constant. However, I let it evolve endogenously as a robustness exercise in order to illustrate the role of firm dynamics in the transition of the post COVID-19 economy.

functions and distributions are equal across periods. See Appendix C for details on how I solve for both equilibria.

Shocks and Government Intervention

I simulate the economic effects of the COVID-19 pandemic through two channels. First, I model the fall in aggregate demand as a fall in the price of the final good.[9] Second, I introduce a labor utilization constraint to reproduce the effects of stay at home orders for workers whose tasks cannot be carried out off-site in non-essential activities: lockdowns meant to contain the spread of COVID-19 prevented significant proportions of workers from reaching their workplaces, while in principle maintaining firms' financial obligations towards them intact due to the government-imposed layoff ban. The two largest government policies put in place in Italy to counteract the strain that either shock put on firm liquidity were government subsidized furloughs (known by their Italian acronym CIG) and long run debt moratoria (henceforth MOR).[10] The former lowered the burden of paying for underutilized labor, while the latter postponed long run debt outlays by a calendar year.

Given that the shocks I study and the subsequent policy responses were completely unexpected and (hopefully) temporary, I model their effects in the following paragraphs, separately from the steady state behavior described above.

Government Subsidized Furloughs (CIG) I model the effect of stay at home orders as an unexpected, temporary constraint on the amount of labor firms can utilize.[11] In the period in which the COVID-19 shock hits, firms initially hire labor $l(k,\varepsilon)$ as described in the previous paragraphs, while only being able to use up to a portion λ of that labor to produce. While firms are free to utilize any quantity of labor l_s up to $\lambda l(k,\varepsilon)$, they are liable for their entire initial payroll $wl(k,\varepsilon)$. The government might decide to intervene nonetheless by paying a portion χ of furloughed workers' salaries. Equation 15 illustrates

[9]This analogous to assuming aggregate demand is perfectly inelastic from the point of view of perfectly competitive firms.

[10]The largest government intervention by far was the introduction of guaranteed loan schemes. These however were designed to provide liquidity for firms, whereas CIG and MOR were designed to reduce firm liquidity needs.

[11]What happened in reality was that workers pertaining to sectors not deemed as essential were not allowed to attend their workplaces for the duration of the lock down. Given I calibrate the model to an annual frequency, assuming this to be analogous to restricting the quantity of labor being utilized within the year amounts to implicitly assuming the absence of costs from interrupting production during a certain time span.

the cash flow firms obtain under CIG.[12]

$$
\begin{aligned}
CF_s\left(l_s, m, k, \varepsilon\right) = p\varepsilon\left(l_s^{\alpha} k^{\beta} m^{\zeta}\right)^{\nu} &- \left(wl_s + p_m m + c_f\right)\left(1 + r_s\right) \\
&- \chi w\left(l\left(k, \varepsilon\right) - l_s\right)\mathbb{1}\left\{l\left(k, \varepsilon\right) > l_s\right\}\left(1 + r_s\right) \\
&l_s \leq \lambda l\left(k, \varepsilon\right)
\end{aligned} \tag{15}
$$

Long-run Debt Moratorium (MOR) My model focuses on short-run debt that firms take on in order to finance their ongoing operations. At the time of choosing inputs, and thus determining liquidity needs, firms perceive long run debt taken on in the past, as well as its current cost, as given. Hence, I model long run debt service as a portion r_l of fixed costs. By postponing long run debt service payments by a calendar year, MOR effectively lowers fixed costs in the current period by $c_f r_j$.

$$
CF\left(l, m, k, \varepsilon\right) = p\varepsilon\left(l^{\alpha} k^{\beta} m^{\zeta}\right)^{\nu} - \left(wl + p_m m + c_f - c_f r_l\right)\left(1 + r_s\right) \tag{16}
$$

Equation 16 shows the cashflow of a firm after the introduction of MOR. When the COVID-19 shock hits, fixed costs are reduced, which lowers the bankruptcy threshold for firms while leaving their input choices unchanged. Producing firms that decide to invest in capital for the following period anticipate that they will have to fulfill the long run debt payment originally corresponding to the current period at the debt's maturity.

$$
\begin{aligned}
V_i\left(k, \varepsilon\right) = \max_{k'} &- \left(k' - \left(1 - \delta\right)k\right) - c_a\left(\left(1 - \delta\right)k, k'\right) + \frac{1}{1 + r} E_{\varepsilon'|\varepsilon} V\left(k', \varepsilon'\right) \\
&- P_{N_l}\left(k', \varepsilon\right)\frac{\left(1 - \tau\right)c_f r_l}{\left(1 + r\right)^{N_l}}
\end{aligned} \tag{17}
$$

Equation 17 shows the modified value of surviving an additional period, where I denote long run debt maturity N_l and the probability of surviving N_l periods $P_{N_l}\left(k', \varepsilon\right)$.

3 Calibration

In this section I calibrate the model economy's ergodic steady state equilibrium to replicate the size distribution, exit dynamics, and aggregate asset holdings of Italian firms. I divide the parameters of the model into those whose values I set outside of the solution of the model, and those that I calibrate endogenously.

[12] The indicator function in Equation 15 prevents double accounting of wages for firms in sectors in which the final good price increases.

Table 1: Calibrated parameters

Parameter		Value
Exogenously Picked Parameters		
Interest rate on liquidity	r_s	0.0318
Discount rate	r	0.0396
Corporate income tax	τ_c	0.275
Payroll tax	τ_w	0.3
Labor Elasticity of Output	α/γ	0.359
Capital elasticity of output	β	0.3
Span of control	ν	0.8
Depreciation rate	δ	0.1
Proportional adjustment cost	c_0^a	$11e-5$
Convex adjustment cost	c_1^a	0.03141
Long run debt costs to fixed costs	r_l	0.076
Average residual maturity on loans	N_l	5
Endogenously Calibrated Parameters		
Fixed Cost of Production	c_f	0.01
Proportional liquidation cost	c_l	0.02
Permanent TFP Stdev.	σ_θ	1.43
Transitory TFP Innovation Stdev.	σ_ε	0.84
Transitory TFP Persistence	ρ	0.1
Mass of Potential Entrants	M_0	$47e3$

I assume all prices are constant and equal to 1 in the ergodic steady state. Panel A in Table 1 shows the parameters whose corresponding values I choose exogenously. I set the interest rate on liquidity to match the average interest rate on short term loans to non-financial businesses in Italy for the period 2006-2018 as reported by the European Central Bank. Similarly, I use the average composite cost of borrowing for long term loans to both households and non-financial corporations as the discount rate. I set the corporate income tax rate to the statutory national level rate reported by OECD for Italy, and the payroll tax rate to that reported in OECD (2018). As shown in Appendix B.2, I identify the labor and intermediate input elasticities of output directly in the data by using the first order conditions from Equation 2 and the assumption regarding the sum of individual input elasticites of output. I take the parameters determining the capital elasticity of output, span of control, capital depreciation, and capital adjustment costs directly from Clementi and Palazzo (2016). In order to estimate the portion of fixed costs due to long-run financing costs, I need to construct an estimate of the latter, which I don't directly observe. Given long-run debt represents 26.5% of Italian firms' liabilities on average in my firm balance sheet data, I attribute that portion of financial costs to long run debt and use it to measure its corresponding ratio with respect to fixed costs.[13] Finally, I obtain the average residual maturity on loans to Italian non-financial corporations covered by COVID-19 guarantees from AnaCredit.[14]

Panel B in Table 1 details the parameters that I calibrate endogenously, as well as their chosen values. These parameters can be roughly divided into three categories: the fixed cost of production and the proportional liquidation cost, which determine the ability firms have to produce positive cash-flow, as well as their incentives to produce in any given period vis-à-vis exiting the market; the standard deviation of both permanent and transitory productivity, as well as the persistence of the latter, which directly affect the firm size distribution resulting in equilibrium; and the mass of potential entrants uniquely pins down the aggregate size of the economy in the ergodic steady state.

Table 2: Calibration results

Target	Data	Model
Exit rate	7.3%	7.3%
Share of bankrupt exiters	18%	18%
Log revenue stdev.	1.24	1.24
Revenue growth stdev.	0.44	0.44
Log revenue autocorrelation	0.92	0.94
Aggregate net worth	$905{\cdot}10^9$	$905{\cdot}10^9$

I choose to calibrate the model to the moments displayed in Table 2. They are informative of the dynamics, size distribution, and aggregate size of Italian firms. I obtain all moments from the official Italian firm registry, as processed and delivered by CERVED. The microdata refer to all manufacturing firms for the years 2006 to 2018. The model generally does a good job of replicating the proposed targets and only struggles to match the autocorrelation of firm revenue.

4 Quantitative Exercise

Using the calibrated model economy I explore the effects of implementing the two types of policies designed to alleviate firms' liquidity shortages described earlier: government subsidized furloughs (CIG) and long-run debt moratorium (MOR).[15] I proceed in various steps. First, I design two unexpected, temporary shocks to the economy: a fall in the

[13]I estimate fixed costs to be the sum of the costs of long-run debt and all costs related to services and usufruct of third party goods.

[14]I thank Enrico Sette for providing me with the statistic.

[15]For further details on both policies see Lo Bello (2020) and Orlando and Rodano (2020).

price of the final good, which stands for the fall in aggregate demand caused by the COVID-19 pandemic, and a tightening of the labor utilization constraint, which recreates the consequences to firms of stay at home orders issued to workers during the height of the pandemic. Next I simulate the effects of said shocks on an economy in which the government does not intervene. Finally, I rerun the simulation assuming the government does introduces each one of the aforementioned policies. I assume the price of the final good evolves geometrically henceforth and takes three years to return to its original level, while the labor utilization constraint returns to its original level within one period. Firms do not initially anticipate a change in either variable, but have perfect foresight regarding their trajectories after the initial shock.

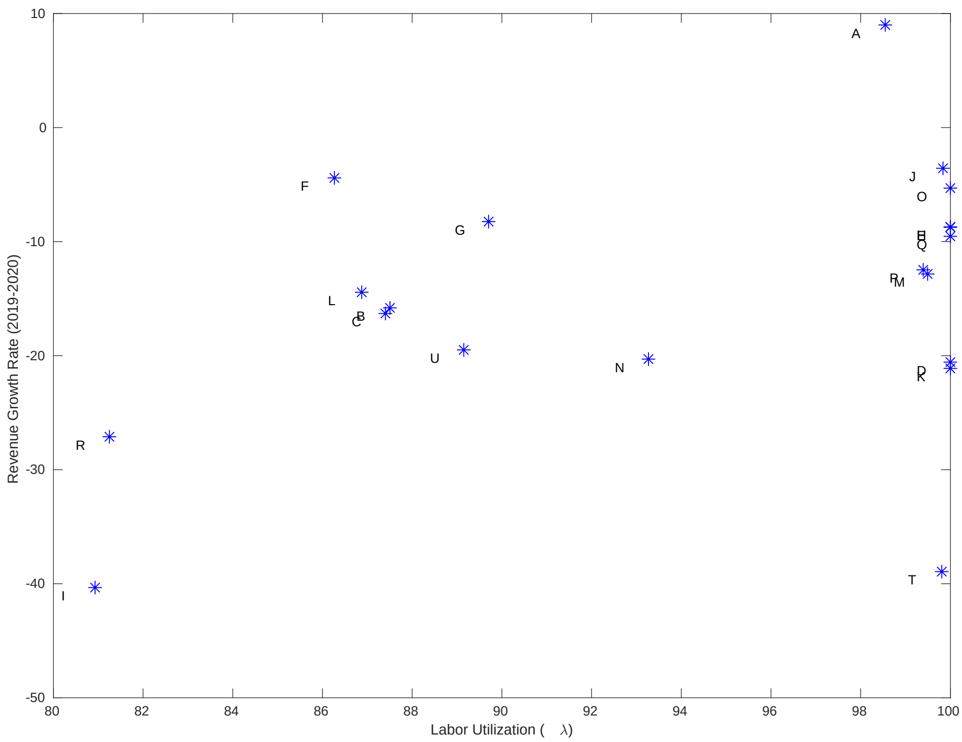


Figure 4: Revenue growth and labor utilization in 2020

I use sectoral data on Italian firms and the impact the COVID-19 pandemic had on them to determine the size of the shocks I incorporate into the simulation of my economy. I employ an estimate of the percentage of workers ordered to stay at home during 2020, and assigned to tasks not conducive to working remotely, at the same two-digit sector level taken from Basso et al. (2020).[16] Additionally, I use pooled microdata from different

[16]I compute the labor utilization index in each sector as one minus one quarter of the percentage of

sources to estimate the fall in revenues during 2020 at the two-digit sector level.[17] Figure 4 compares the incidence of stay at home orders and the fall in revenues in my data.[18]

In order to understand the properties of the policies I study, I simulate the economy under four scenarios representing four sectors of the economy diversely hit by the two shocks I consider: Sector J (Information and Communication) suffered a moderate limit to its labor utilization, as well as a moderate fall in revenues; Sector F (Construction) saw its workforce severely limited during lockdowns, yet a moderate fall in revenues; Sector T (Activities of Households as Employers) was barely affected by workers' stay at home orders, yet suffered a major fall in revenues; and Sector I (Accommodation and Food Service Activities), which was hit hard by both shocks. I pick the magnitude of each shock as follows: First, I set λ, the labor utilization constraint, to the index shown in Figure 4; then I back out the price of the final good needed to endogenously produce the corresponding fall in revenues in an economy in which the government intervenes both through CIG and MOR.

The exercise I run has three parts: first, the economy starts in the steady state calibrated above, which represents the Italian economy pre-pandemic; then, the price of the final good falls unexpectedly; next, the price slowly returns to its original level; finally, once the price is at its original level, the economy evolves towards its original steady state.

Moderate Labor Utilization Fall, Moderate Aggregate Demand Fall In the first scenario, firms suffer a minor fall in labor utilization ($\Delta\lambda = -0.17\%$) as well as a slight drop in the price of the final good ($\Delta p = -1.39\%$). Table 3 summarizes the effect of the shocks on impact. In an economy without government intervention, the bankruptcy rate increases slightly, while a significantly higher portion of firms decide to endogenously exit the market. The entry rate on the other hand falls to a very minor degree and there is a significant fall in the stock of firms. These dynamics lead to significant employment, capital stock, and output falls, which are partially contained by the positive selection at exit evidenced by the increase in average firm productivity. Although the higher Olley Pakes covariance signals a reduction in misallocation, the fact that both unweighted and weighted average productivity evolve similarly suggests that the resulting productivity gains are small.[19] On the financial side, decreased production leads to lower overall

workers employed in non-essential activities not able to work remotely.

[17] Given data availability, I use the growth rate of reported revenues between January through November 2019 and January through November 2020.

[18] I use the Statistical Classification of Economic Activities in the European Community, NACE Rev. 2 to classify industries. See https://ec.europa.eu/eurostat/documents/3859598/5902521/KS-RA-07-015-EN.PDF for more information.

[19] As shown in Appendix B.4, misallocation in this economy stems from firms not knowing their productivity at the time of investing in capital. Misallocation improves after the shock because firms

borrowing and a fall in profits. Given the situation, firms prefer to disburse dividends and drastically cut their investment.

Table 3: Percentage deviation from steady state on impact ($\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$)

	Shock	*CIG*	*MOR*		*Shock*	*CIG*	*MOR*
Firm Dynamics				*Corporate Finance*			
Bankruptcy Rate*	0.97	0.96	-0.04	Aggregate Borrowing	-6.27	-7.27	-3.74
Endogenous Exit Rate*	6.53	6.54	1.61	Aggregate Profits	-6.28	-6.19	-0.89
Entry Rate*	-0.07	-0.07	0.08	Aggregate Dividends	68.88	68.99	27.62
Number of Firms	-7.98	-7.98	-1.5	Aggregate Investment	-100.62	-100.62	-37.43
Real Outcomes				*Firm Selection and Resource Allocation*			
Aggregate Employment	-4.18	-7.03	-0.78	Unwghtd. Avg. TFP	7.69	7.69	2.05
Aggregate Capital	-7.88	-7.88	-1.45	Wghtd. Avg. TFP	7.68	7.68	1.99
Aggregate Output	-6.22	-7.03	-2.90	Olley Pakes Covariance	5.96	5.96	-5.47

* Percentage points

Subsidizing temporary furloughs in this scenario has a limited effect on firm dynamics: it helps slightly lower bankruptcy, but incentivizes most of the rescued firms to voluntarily exit. Similarly, it does not have a significant effect on the entry rate. All in all, the composition and number of firms is unchanged. Subsidizing furloughs does incentivize firms to use less of their initially hired workforce, thus lowering aggregate employment and output further than in the no-intervention case.[20] Aggregate borrowing correspondingly decreases with respect to the absence of CIG, although aggregate profits and dividends are higher.

The introduction of long-term debt moratoria significantly improves aggregate outcomes in this scenario. By relieving firms of a portion of their fixed costs, MOR lowers the bankruptcy rate with respect to the initial steady state, while significantly reducing the rate of voluntary liquidation with respect to an economy in which the government does not intervene. Additionally, the entry rate increases with respect to the initial steady state. Overall, MOR dampens the fall in the number of firms, as well as the increase in average productivity. Relatively less-productive firms benefit relatively more from the fall in fixed costs, which leads to an increase in misallocation. In spite of the lower average productivity of firms, their greater number has a positive effect on aggregate employment, capital, and output. This on the other hand softens the fall in aggregate borrowing and profits, as well as investment, while halving the increase in dividend disbursements.

with higher marginal productivity of capital are more prone to exit. This property of the model is uniform across the four shock configurations I present.

[20] In reality, workers under CIG did not seize to be employed, yet were not being utilized by their employers. The model however does not make a distinction between both. The fall in employment due to the introduction of CIG in the model, should thus be interpreted as a fall in utilization.

Summing up, the introduction of CIG has a limited effect in this scenario, other than reducing employment and output, while marginally increasing profits and firm valuation. The reason is that it barely reduces firms' liquidity needs. MOR on the other hand helps lower-productivity firms reduce their immediate outlays, thus preventing a wave of bankruptcies and sustaining aggregate employment and output.

Sharp Labor Utilization Fall, Moderate Aggregate Demand Fall This scenario, summarized in Table 4, represents sectors of the economy for which a significant portion of workers were under stay at home orders ($\Delta\lambda = -13.73\%$) but aggregate demand only fell slightly ($\Delta p = -0.22\%$). The overall effects of this combination of shocks in the absence of government intervention go in the same direction as for the first scenario and are similar in magnitude, although the fall in aggregate employment is relatively larger when compared to the fall in the other real outcomes of the model.

Table 4: Percentage deviation from steady state on impact ($\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$)

	Shock	*CIG*	*MOR*		*Shock*	*CIG*	*MOR*
Firm Dynamics				*Corporate Finance*			
Bankruptcy Rate*	1.55	0.76	-0.04	Aggregate Borrowing	-9.99	-5.44	-6.63
Endogenous Exit Rate*	5.95	-0.05	0.10	Aggregate Profits	-8.78	-1.04	-3.18
Entry Rate*	-0.07	0.02	-0.03	Aggregate Dividends	52.23	5.78	-1.85
Number of Firms	-7.98	-0.68	0.03	Aggregate Investment	-82.93	-9.24	-2.45
Real Outcomes				*Firm Selection and Resource Allocation*			
Aggregate Employment	-17.20	-13.87	-13.56	Unwghtd. Avg. TFP	7.69	1.30	0.73
Aggregate Capital	-7.88	-0.60	0.03	Wghtd. Avg. TFP	7.68	1.30	0.79
Aggregate Output	-8.33	-4.64	-4.30	Olley Pakes Covariance	5.96	1.73	9.57

* Percentage points

The introduction of CIG in this scenario has an important impact on firm dynamics: the fall in the bankruptcy rate is cut in half on impact, the endogenous exit rate falls slightly below its steady state level, and the entry rate increases above its original level. Overall, most of the firms that would have exited without government intervention end up staying open. As a consequence, the fall in aggregate output is cut by half and aggregate capital barely falls. In spite of the incentive to disengage employees from their tasks given by subsidizing furloughs, aggregate employment is slightly higher compared to when the government does not intervene. This in turn softens the fall in aggregate borrowing, profits, and investment, while simultaneously reducing firms' incentives to disburse dividends.

MOR once again has an important effect on the different outcomes of the economy. Most of the policy's effect on dynamics works through the bankruptcy rate, which falls below its original steady state level, although the spike in liquidation and the fall in the entry rate are also dampened. As a consequence, the number of firms increases with respect to its original level. This comes at a cost to the average productivity of operating firms, which increases only half as much with respect to the original steady state as it does under CIG. Although the effects on aggregate employment and output are similar to those under the latter policy, the introduction of long-run debt moratoria manages to increase the stock of capital with respect to the original steady state. On the financial side, borrowing and profits are slightly lower compared to those achieved by CIG, while aggregate dividends are further reduced, and aggregate investment closer to reaching its steady state level.

Under the scenario of a significant fall in labor utilization, but only a minor fall in aggregate demand, CIG becomes a powerful tool by allowing firms to drastically reduce their fixed payroll commitments to workers that are unable to produce. MOR is once again an effective policy tool under this scenario, yet again at a cost to selection at exit compared to CIG.

Moderate Labor Utilization Fall, Sharp Aggregate Demand Fall Table 5 shows the effects on impact of a sector with high labor utilization ($\Delta\lambda = -0.19\%$) and a strong fall in aggregate demand ($\Delta p = -10.4\%$). All variables move in the same direction as in the other scenarios in the absence of government intervention, although the magnitudes of their movements are noticeably larger than in the first two scenarios.

Table 5: Percentage deviation from steady state on impact ($\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$)

	Shock	*CIG*	*MOR*		*Shock*	*CIG*	*MOR*
Firm Dynamics				*Corporate Finance*			
Bankruptcy Rate*	5.61	5.59	3.16	Aggregate Borrowing	-33.64	-39.35	-34.67
Endogenous Exit Rate*	28.21	28.23	30.66	Aggregate Profits	-42.42	-41.47	-39.89
Entry Rate*	-0.16	-0.16	-0.16	Aggregate Dividends	305.23	306.35	308.20
Number of Firms	-36.32	-36.32	-36.32	Aggregate Investment	-468.48	-468.48	-468.48
Real Outcomes				*Firm Selection and Resource Allocation*			
Aggregate Employment	-23.62	-40.39	-23.62	Unwghtd. Avg. TFP	39.07	39.07	39.07
Aggregate Capital	-36.02	-36.02	-36.02	Wghtd. Avg. TFP	38.81	38.81	38.81
Aggregate Output	-36.00	-40.39	-36.00	Olley Pakes Covariance	0.91	0.91	0.91

* Percentage points

Similarly to the first scenario, introducing government subsidized furloughs produces very marginal effects on firm dynamics in this scenario: it slightly reduces bankruptcy, while increasing voluntary liquidation. Consequently, average productivity remains unaffected. On the other hand, aggregate employment is strongly reduced, as firms are incentivized to furlough a significant portion of their work forces, and aggregate output follows suit to a lower degree. As a result, aggregate borrowing is lower compared to an economy without government intervention, and aggregate profits and dividends are slightly higher. Aggregate investment remains unaltered.

The introduction of MOR falls short of delivering relief to the sector in a similar way. Although it considerably lowers bankruptcy, most of the spared firms end up liquidating voluntarily. As a consequence, the total number of firms and their composition is largely unchanged by the policy, as are all real outcomes. Although aggregate borrowing falls slightly, and aggregate profits and dividends rise, aggregate investment stays constant.

In this scenario CIG proves ineffective once more, as reducing firms' payrolls can only come at the expense of reducing production. MOR, on the other hand, while once again a valid tool to reduce the number of bankruptcies among firms, is unable to retain a significant amount of firms in the market due to the grim perspectives the latter face in terms of aggregate demand, and thus future revenues.

Sharp Labor Utilization Fall, Sharp Aggregate Demand Fall The final scenario, illustrated in Table 6, concerns a sector in which both labor utilization ($\Delta\lambda = -19.07\%$) and aggregate demand ($\Delta p = -10.5\%$) fall sharply. All variables react in the same direction as is the case with all other scenarios, while the magnitudes are closest to the previous case, with a sharp drop in aggregate demand.

Table 6: Percentage deviation from steady state on impact ($\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$)

	Shock	*CIG*	*MOR*		*Shock*	*CIG*	*MOR*
Firm Dynamics				*Corporate Finance*			
Bankruptcy Rate*	7.06	5.70	5.72	Aggregate Borrowing	-38.56	-39.41	-39.59
Endogenous Exit Rate*	26.75	28.12	28.10	Aggregate Profits	-48.27	-41.58	-45.72
Entry Rate*	-0.16	-0.16	-0.16	Aggregate Dividends	298.66	306.52	301.65
Number of Firms	-36.32	-36.32	-36.32	Aggregate Investment	-468.86	-468.86	-468.86
Real Outcomes				*Firm Selection and Resource Allocation*			
Aggregate Employment	-38.07	-40.47	-38.07	Unwghtd. Avg. TFP	39.07	39.07	39.07
Aggregate Capital	-36.02	-36.02	-36.02	Wghtd. Avg. TFP	38.81	38.81	38.81
Aggregate Output	-39.79	-40.47	-39.79	Olley Pakes Covariance	0.91	0.91	0.91

* Percentage points

Although the introduction of CIG noticeably lowers the bankruptcy rate, it is unable to prevent firms from choosing to liquidate en masse. Given these exit choices, the rest of the outcomes of the economy closely follow those resulting from the previous scenario. Similarly, MOR does not provide an effective policy tool under these circumstances. In summary, although both policies are effective ways to curb firms' liquidity needs in the short run, the sharp fall in aggregate demand, and the perspective of its slow return to its original level, incentivize firms to liquidate, thus rendering both policies ineffective.

Long Run Dynamics Appendix A contains figures illustrating the long-run behavior of the different aggregates explored above. Figures 6 through 9 show the dynamics of firms across time. The effects of either shock on bankruptcy and voluntary liquidation are short lived: as soon as both shocks dissipate, both exit rates level off at their original steady state levels. Entry rates, on the other hand, remain high as the economy transitions back to its original state. However, the increased influx of firms cannot fully compensate the initial exit wave, thus leading to a slow rebuilding of the aggregate stock of firms.

Figures 10 and 11 illustrate that the effect of both shocks on firm selection is purely temporary: when the shocks hit, the least profitable firms exit. As the final good price and labor utilization return towards higher levels, most of the firms that enter produce with productivity levels that would have warranted their exit under the initial shock. Figure 12 shows that the extremely low level of misallocation in the economy is likewise only affected temporarily by the shocks.

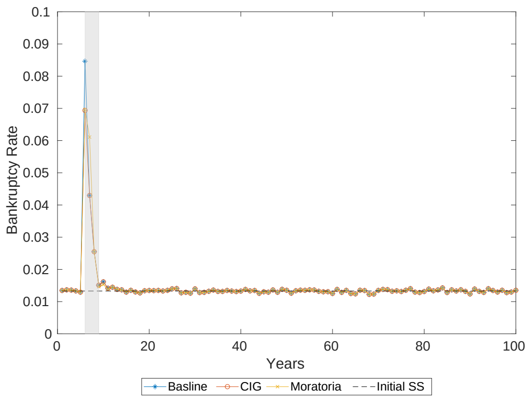
Figures 13 through 15 show the dynamics of real outcomes in the economy. Given the assumed market structure displays constant returns to scale, employment, capital, and output closely follow the dynamics of the stock of firms, thus requiring a significant amount of time post-shock to regain their initial levels. Aggregate borrowing, shown in Figure 16, closely follows the dynamics of employment, as it reflects firms' liquidity needs. Aggregate profits in Figure 17 likewise mimic the dynamics of the former. Figure 18 illustrates that aggregate dividends follow a similar path, albeit after a relatively larger initial reaction to the shocks. Out of all financial variables, aggregate investment is the only one that displays a solely temporary deviation from its initial level, as shown in Figure 19.

In summary, the shocks to labor utilization and the price of the final good only display temporary effects on exit and growth rates in this economy, while leaving long lasting effects on the aggregate level of the real outcomes of the economy by way of the slow recovery of the stock of firms. Prompt, adequately-sized government intervention in this economy is thus vital in order to avoid the scaring effects of a mass exit of firms.

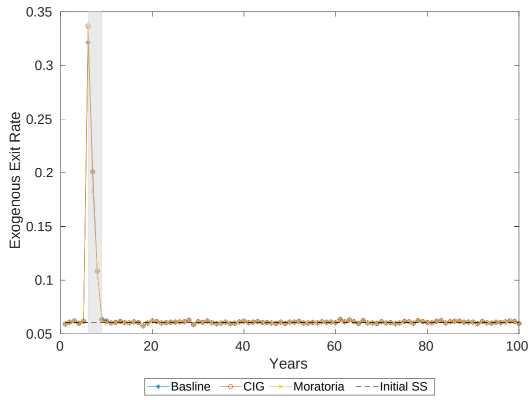
Constant Input Prices and Constant Mass of Entrants Although the long run dynamics of the model are in line with other findings in the literature (see e.g. Clementi and Palazzo, 2016), I consider the outcomes of the baseline model a pessimistic scenario. The lack of input price adjustment and the constant mass of potential entrants in the baseline model directly impact the rate at which the state space returns to its original ergodic steady state distribution. Assuming input prices were endogenous, these might fall in the short-run as the demand for inputs is reduced, thus incentivizing further entry and possibly a more rapid rebuild of the stock of firms.[21] With regard to the mass of potential entrants, the baseline model assumes that entrepreneurs leaving the market never attempt to re-enter in the future. This might indeed be a stark assumption, especially in sectors with low costs of temporary inactivity and subsequent re-entry, thus leading to an increase in the mass of potential entrants in the aftermath of a wave of temporary shutdowns.

Figure 5: Firms Dynamics in a Highly Hit Sector ($\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$) in which Exiters Re-enter

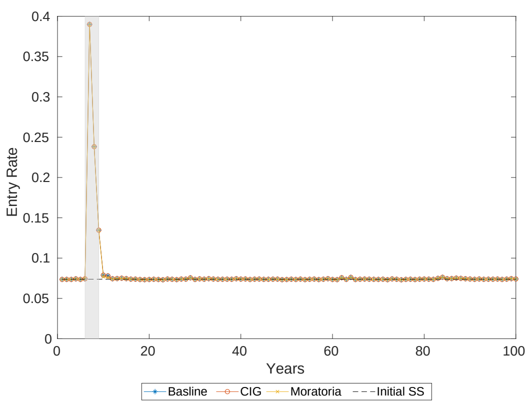
(a) Bankruptcy Rate



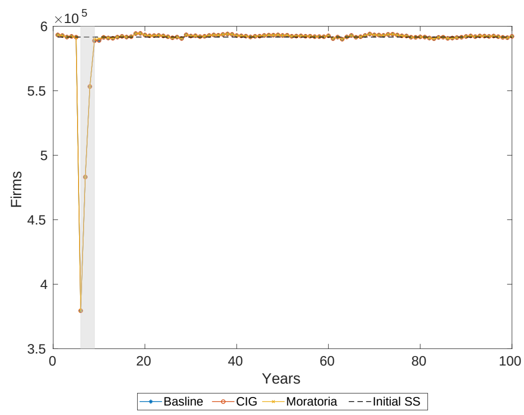
(b) Endogenous Exit Rate



(c) Entry Rate



(d) Number of Firms



The shaded area represents the periods for which final good price p is below its original steady state value.

Figure 5 shows the firm dynamics of a highly-affected sector ($\Delta\lambda = -19.07\%$, $\Delta p =$

[21] Even though this hypothesis currently seems highly debatable, as evinced for example by the world-wide chip shortage underway since late 2020. Additionally, the aforementioned Clementi and Palazzo (2016) do include endogenous prices in their model.

−10.5%) in a version of the model in which the mass of potential entrants $M_{0,t}$ is equal to the value of M_0 calibrated in Section 3 plus the difference between the number of producers at the end of the previous period and the number of producers in the original steady state equilibrium. The COVID-19 shock has no scarring effect in this alternative economy, due to the fact that businesses can re-enter quickly and in a cost-less manner. As an example, think of an owner of a vacation rental business. During the pandemic, the owner goes out of business by not being able to house tourists. However, the minute all travel restrictions are lifted, business can be resumed at a low start-up cost, without having endured significant capital depreciation during the downtime. It would therefore seem reasonable to assume this business to have directly become a potential entrant after having originally exited the market.[22]

Analogously to my interpretation of the baseline model results, I view the alternative version of the model as an overly optimistic scenario. The rate at which exiting firms might re-enter their sectors after the aggregate economy returns to its original state will vary on a sector-by-sector basis, lying somewhere between the two scenarios described above. The model thus suggests that one dimension governments need to pay attention to when allocating funds across distressed sectors, are the costs firms might face when re-entering, which will determine the scarring effects of the COVID-19 shock.

5 Conclusion

I develop a firm dynamics framework in which firms face the need to seek out external liquidity in order to finance their current operations. I simulate an unexpected aggregate demand shock and a temporary drop in labor utilization, which deepens firms' dependence on external liquidity. I show that the model predicts firm exit to increase and firm entry to fall as a consequence of the shock. The dynamics of real variables closely follows the dynamics of firms both in the short- and long run. Although the effects in terms of growth rates vanish after a short number of periods, the shock produces long-lasting effects in terms of levels in the economy.

When exploring the efficacy of government subsidized furloughs and long-run debt relief, I find that the effects of these policies mostly work by reducing fixed costs faced by firms in times of high liquidity needs. By helping firms with lower revenue streams avoid bankruptcy, they can avoid a deep plunge in the number of firms in the economy, thus shortening the time needed for the stock of firms to rebuild, as long as firms' worsened future prospects don't lower their continuation values to the point of incentivizing them

[22] I thank Giacomo Rodano for providing this especially clarifying example.

to voluntarily liquidate. This suggests that some of the support policies put in place so far might need to be continued in sectors taking longer to recover.

Even though my model underlines the importance of avoiding a generalized wave of firm exit which could scar the economy in the long run, several ingredients are missing that might amplify the effect of the policies studied. For one, there is no growth in the model economy. What if avoiding a great number of exits could prevent endogenous growth drivers from plunging, thus sustaining growth in the long run? The dynamics of real variables in the model closely follow the dynamics of firms. What if there were increasing returns to certain industries, how would the recovery dynamics look? Prices are all set exogenously in this model. What if lower input demand could lower input prices, thus alleviating the effects of the aggregate demand shock? How can governments design and sustain cost-effective support schemes in the face of ever declining fiscal space? These are some of the important pieces of the puzzle absent in the model that I leave to future research.

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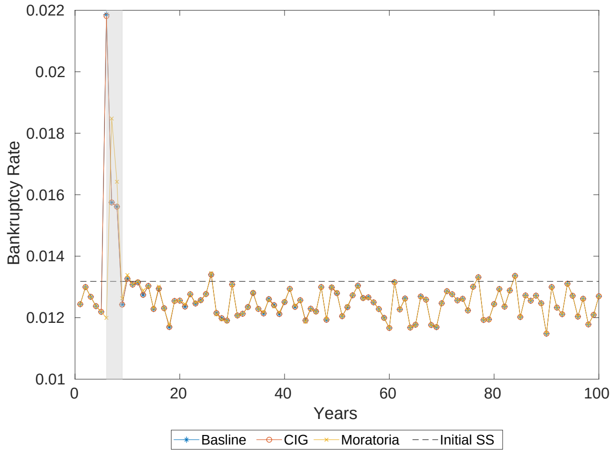
A Additional Figures

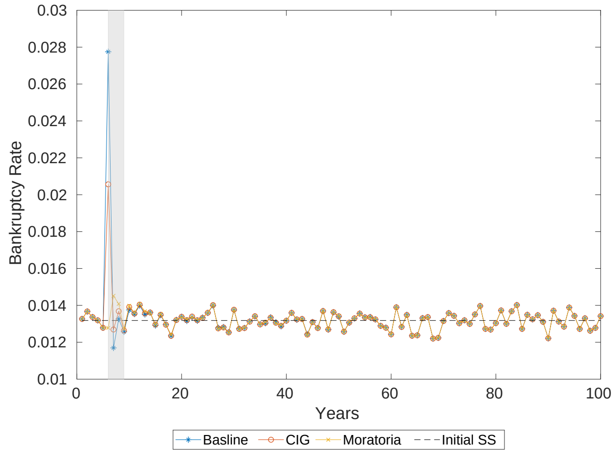
Firm Dynamics

Figure 6: Bankruptcy Rate

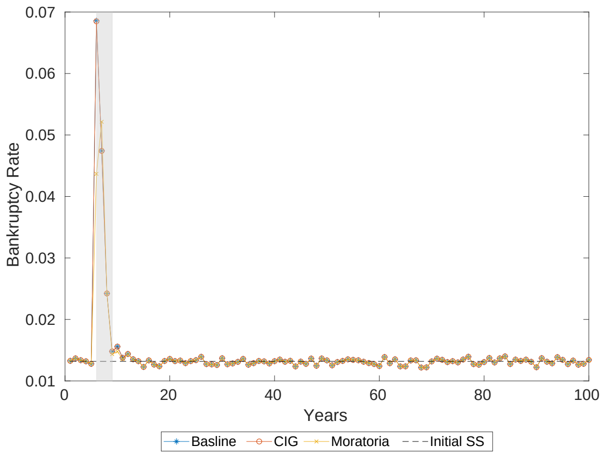
(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$



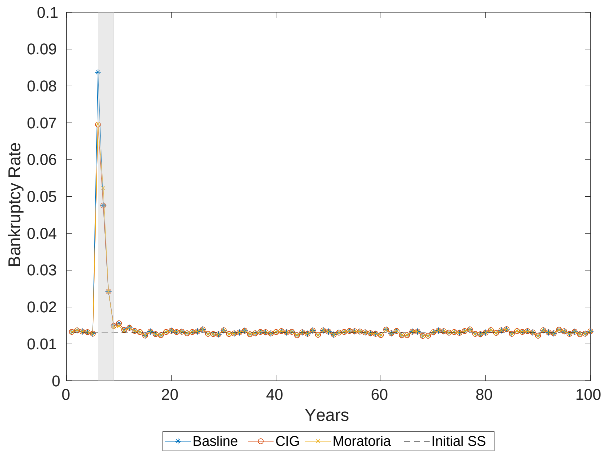
(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$



(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$



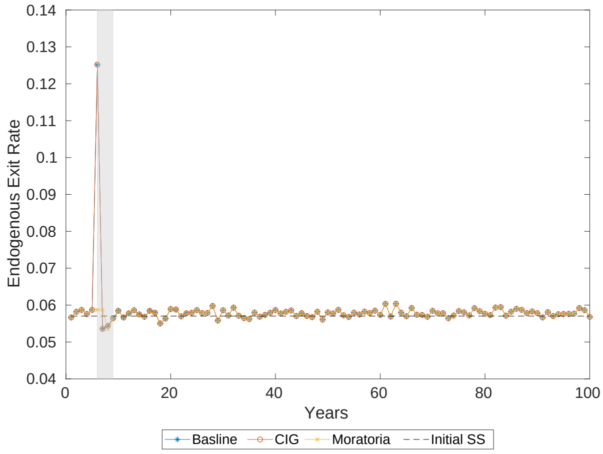
(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$



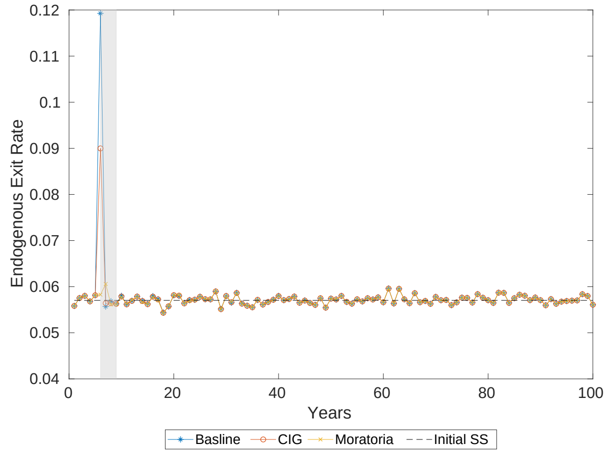
The shaded area represents the periods for which final good price p is below its original steady state value.

Figure 7: Endogenous Exit Rate

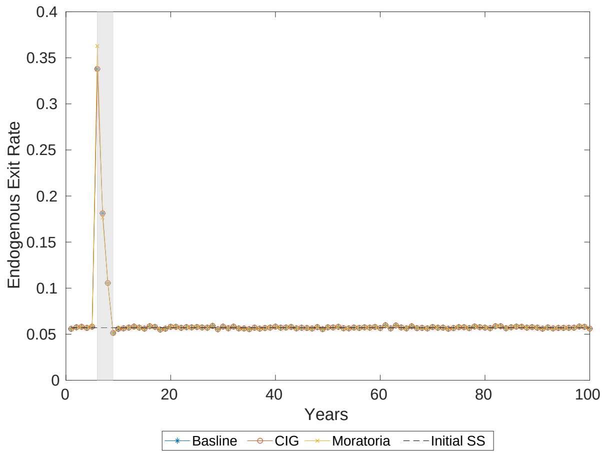
(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$



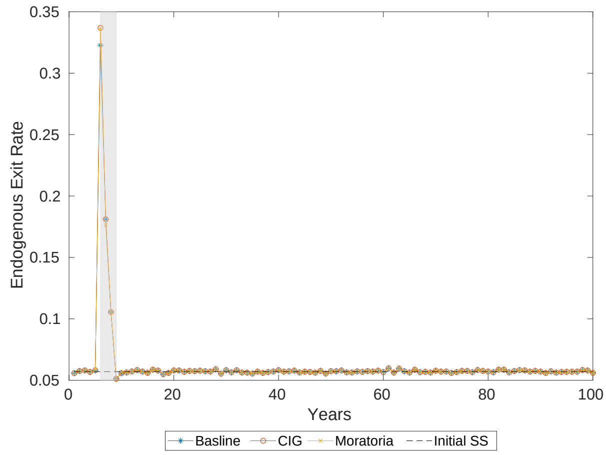
(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$



(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$

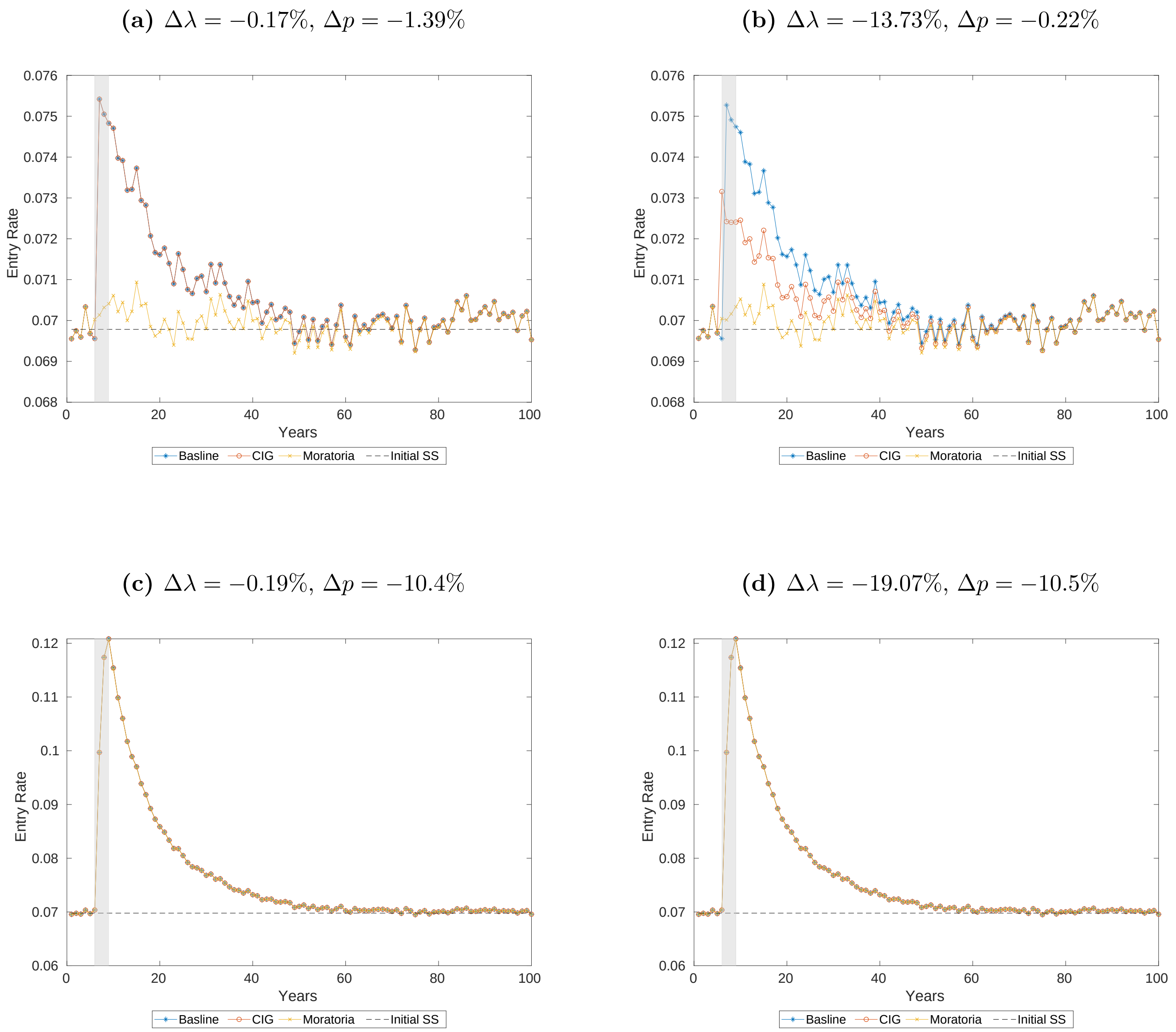


(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$



The shaded area represents the periods for which final good price p is below its original steady state value.

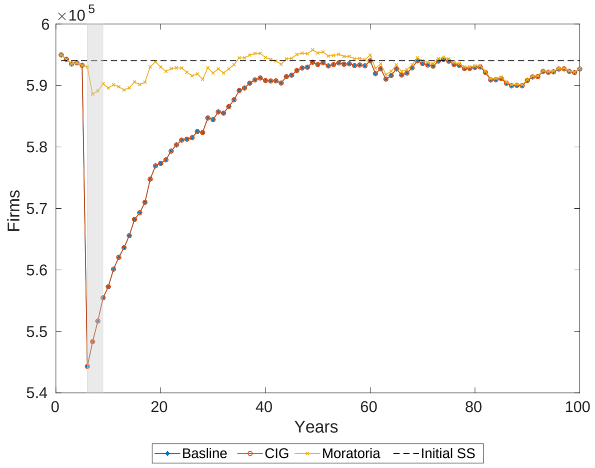
Figure 8: Entry Rate



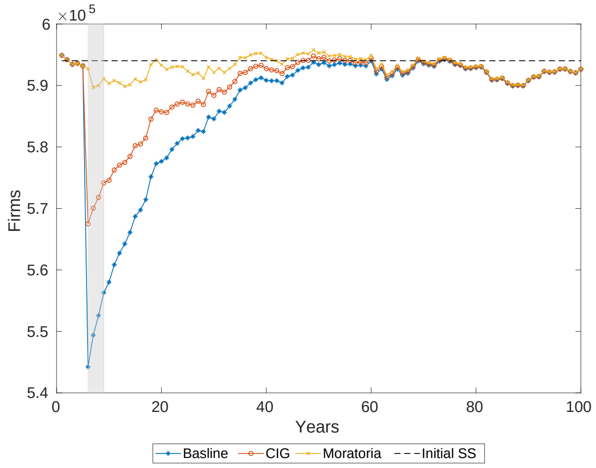
The shaded area represents the periods for which final good price p is below its original steady state value.

Figure 9: Total Number of Producers

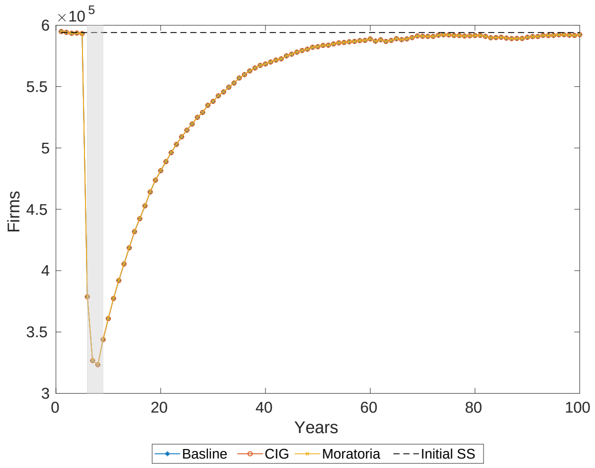
(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$



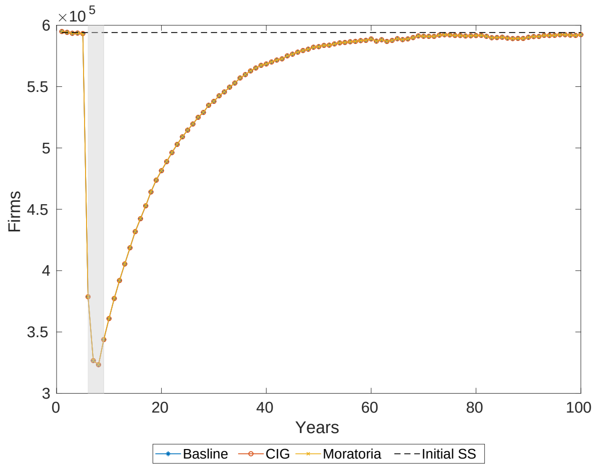
(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$



(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$



(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$

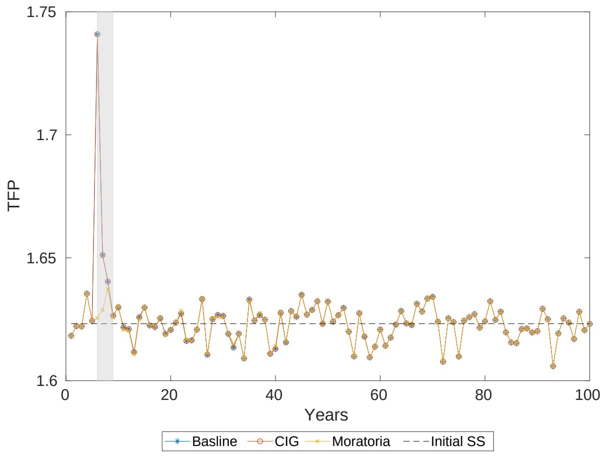


The shaded area represents the periods for which final good price p is below its original steady state value.

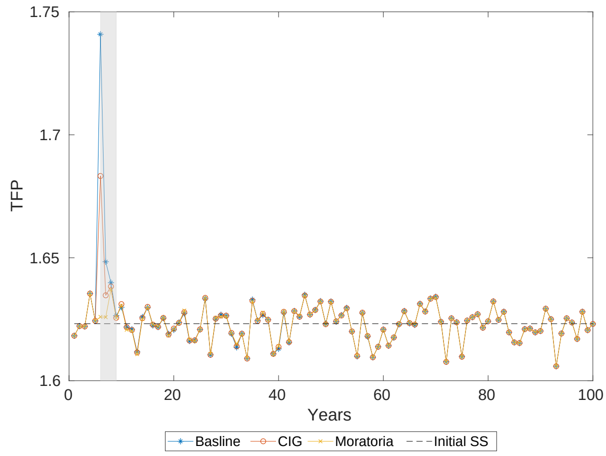
Firm Selection and Resource Misallocation

Figure 10: Unweighted Average TFP

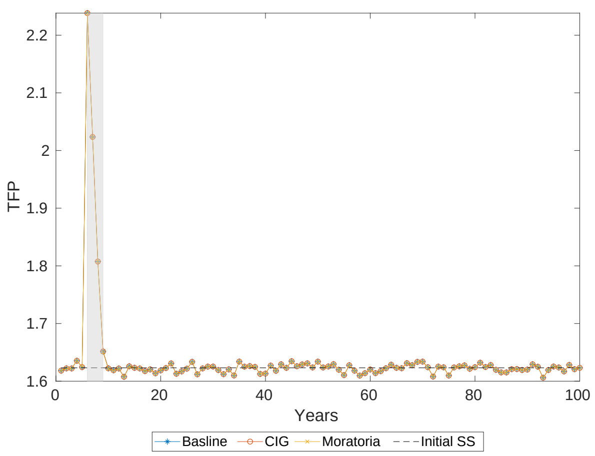
(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$



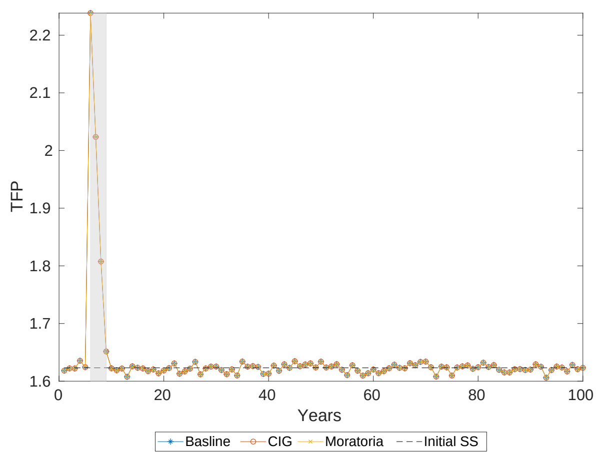
(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$



(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$



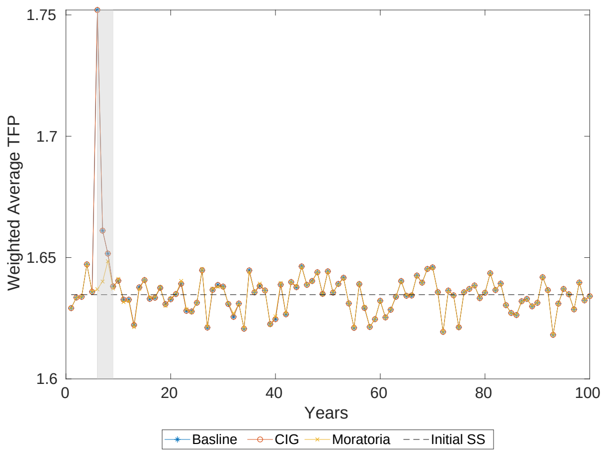
(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$



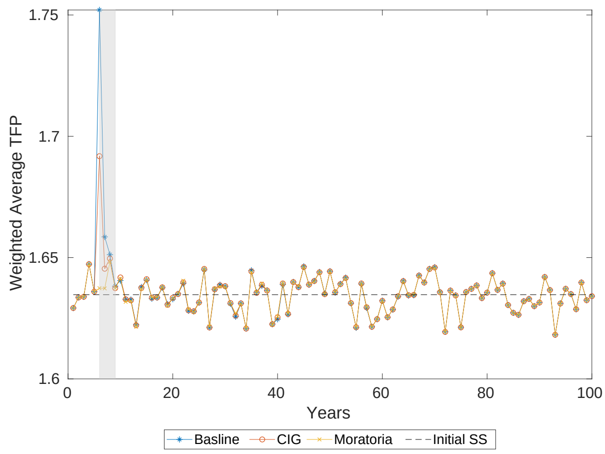
The shaded area represents the periods for which final good price p is below its original steady state value.

Figure 11: Weighted Average TFP

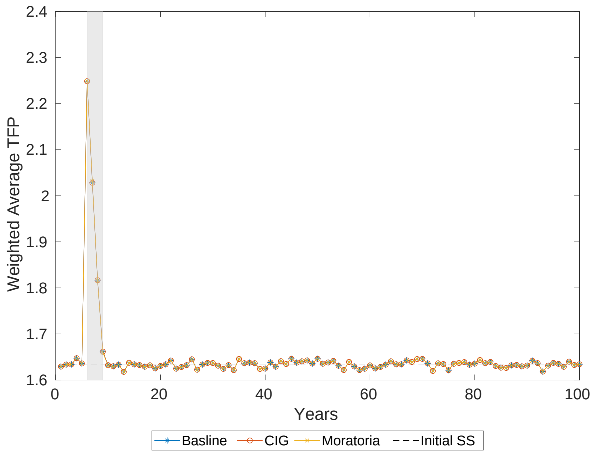
(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$



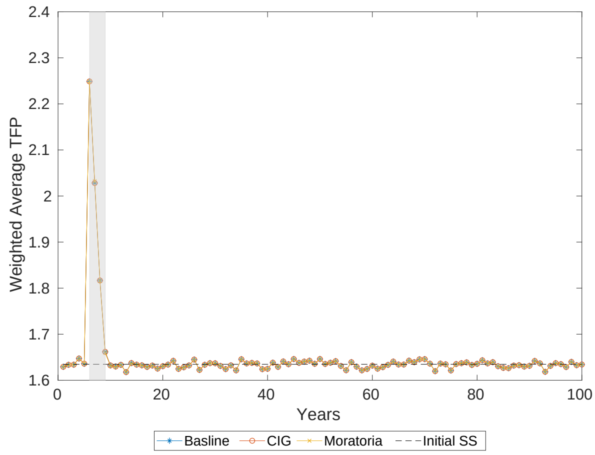
(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$



(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$



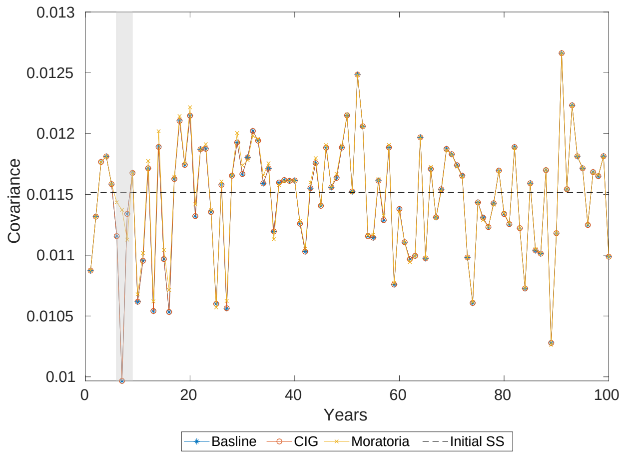
(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$



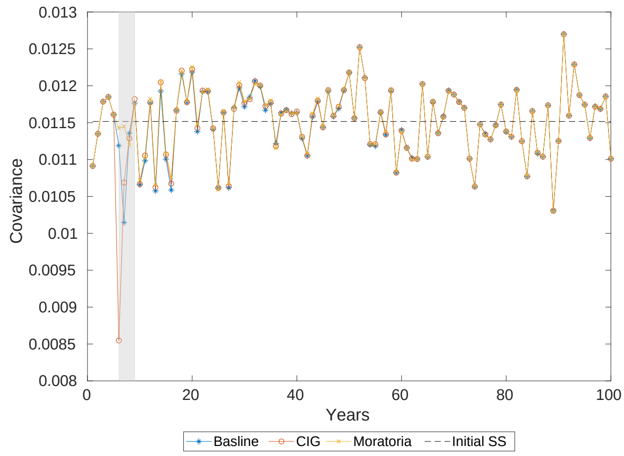
The shaded area represents the periods for which final good price p is below its original steady state value.

Figure 12: Olley Pakes Covariance

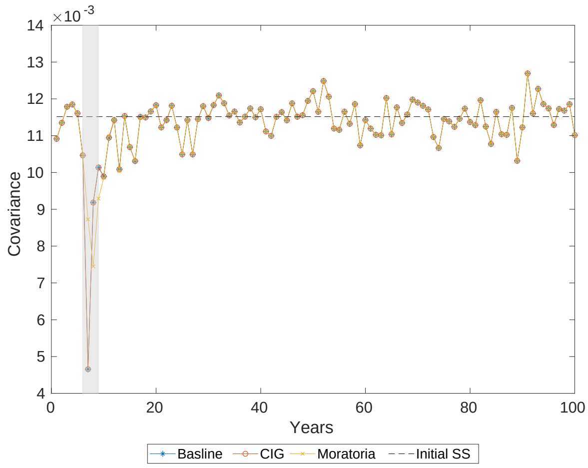
(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$



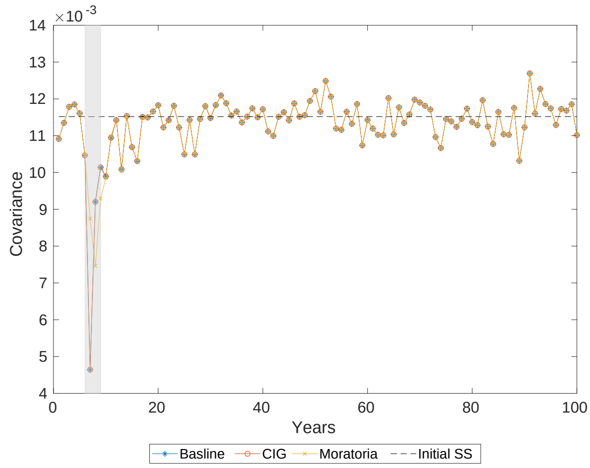
(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$



(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$



(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$

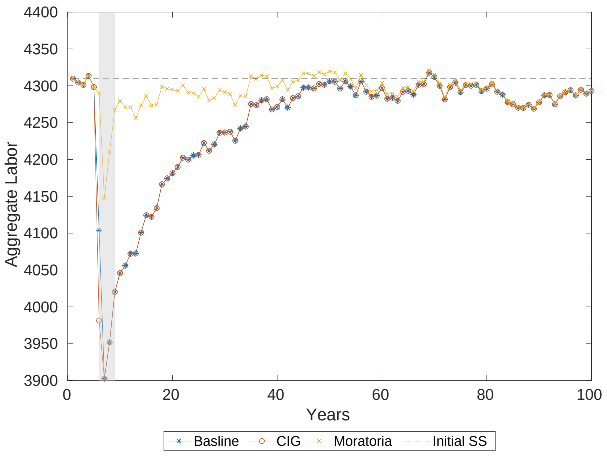


The shaded area represents the periods for which final good price p is below its original steady state value.

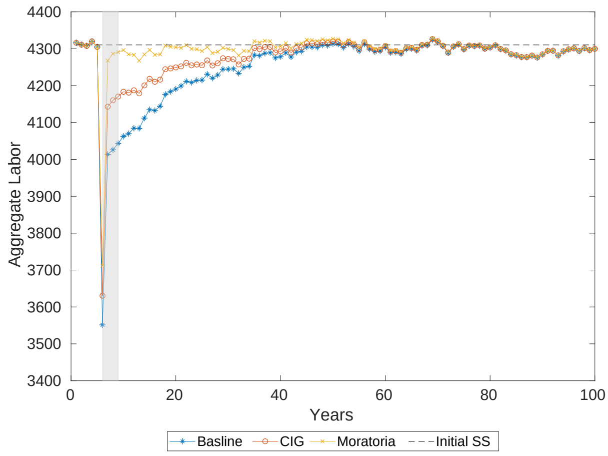
Real Outcomes

Figure 13: Aggregate Employment

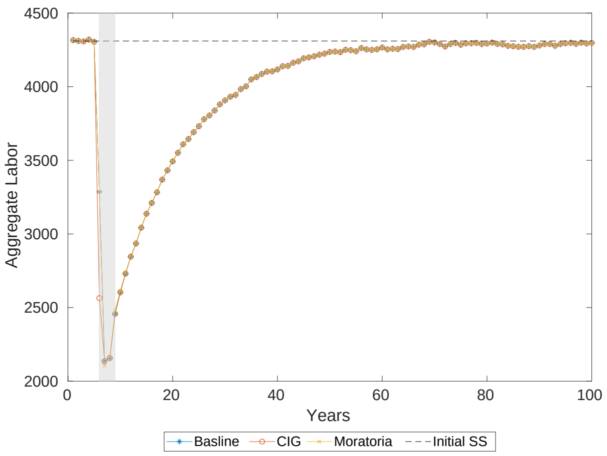
(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$



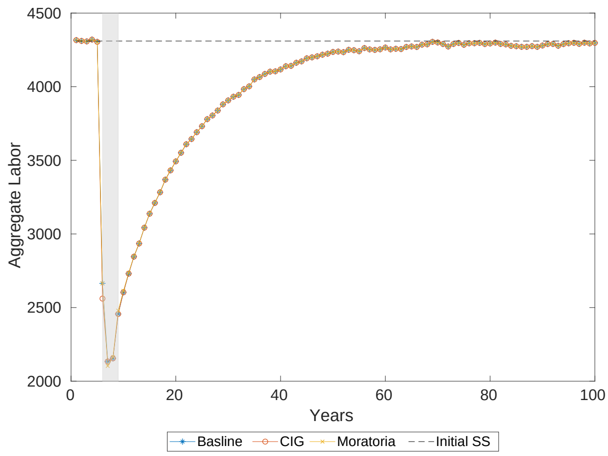
(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$



(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$



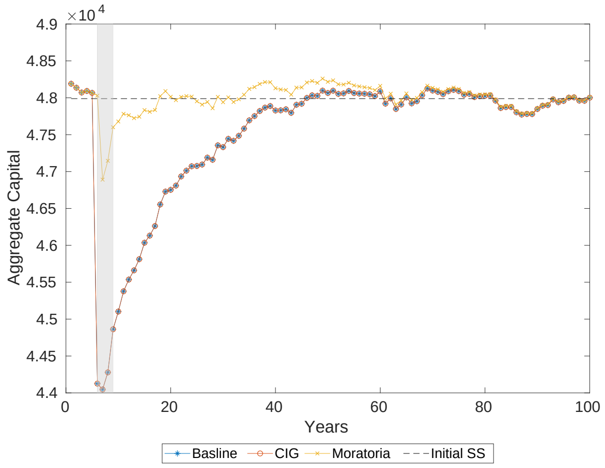
(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$



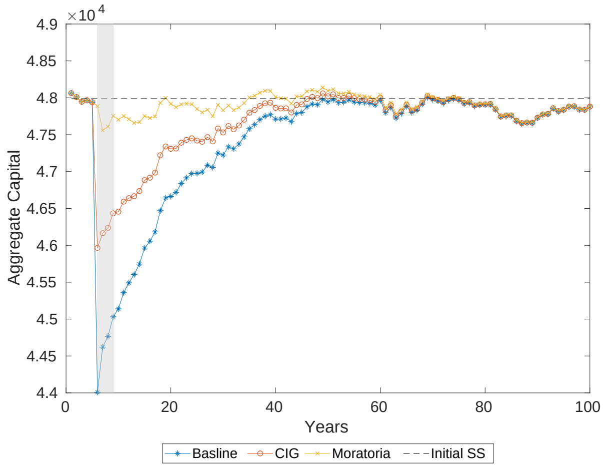
The shaded area represents the periods for which final good price p is below its original steady state value.

Figure 14: Aggregate Capital

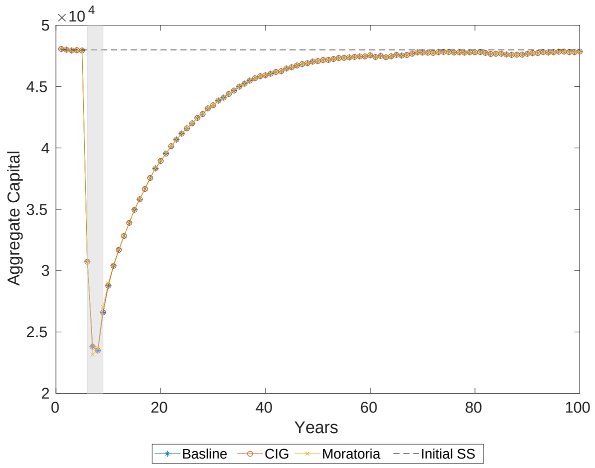
(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$



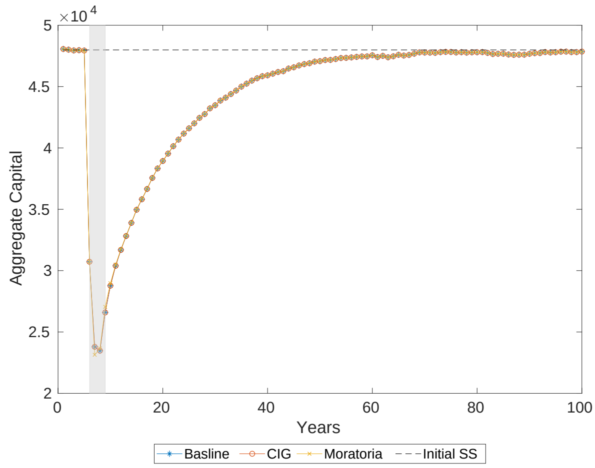
(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$



(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$

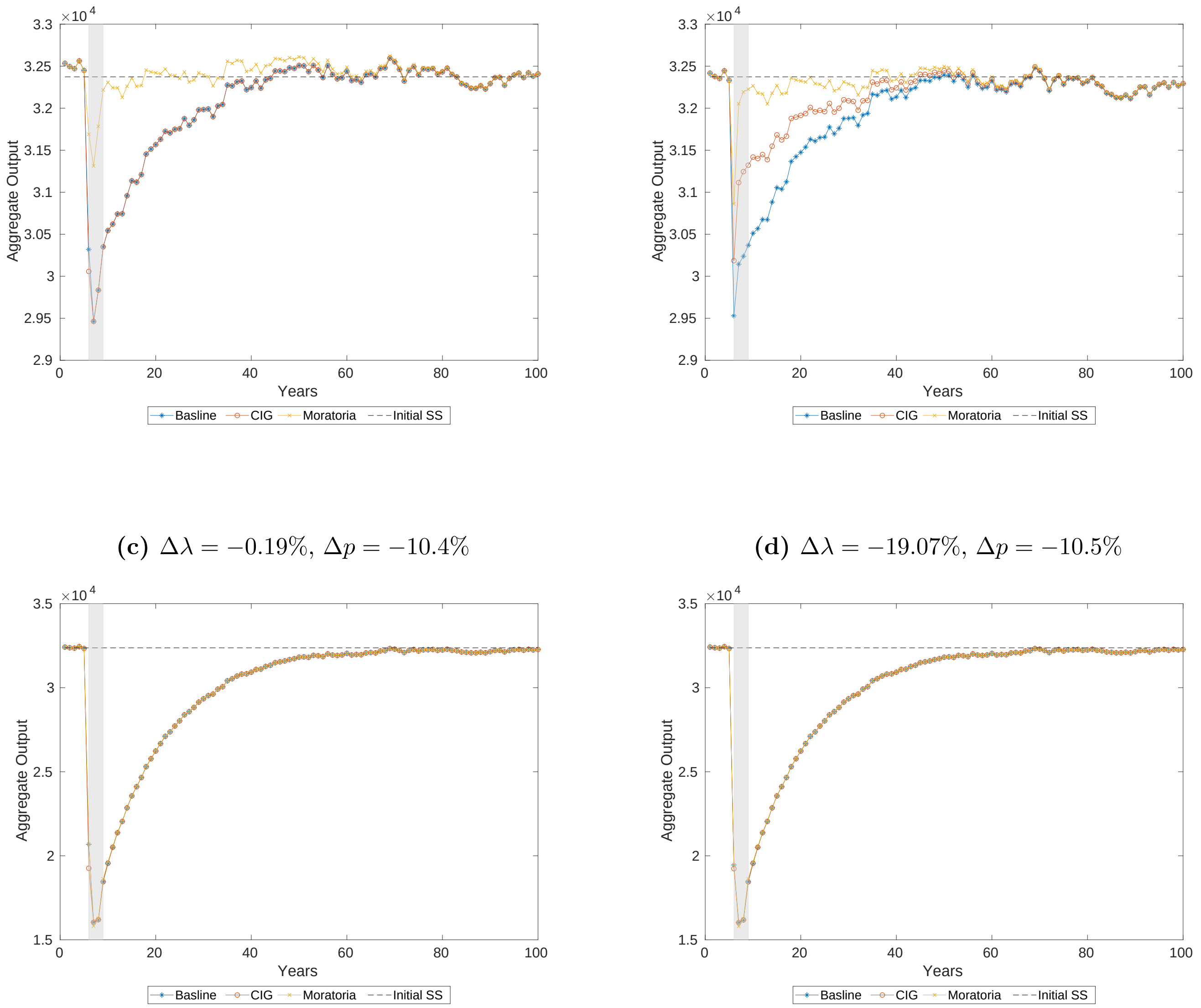


(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$



The shaded area represents the periods for which final good price p is below its original steady state value.

Figure 15: Aggregate Output



(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$

(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$

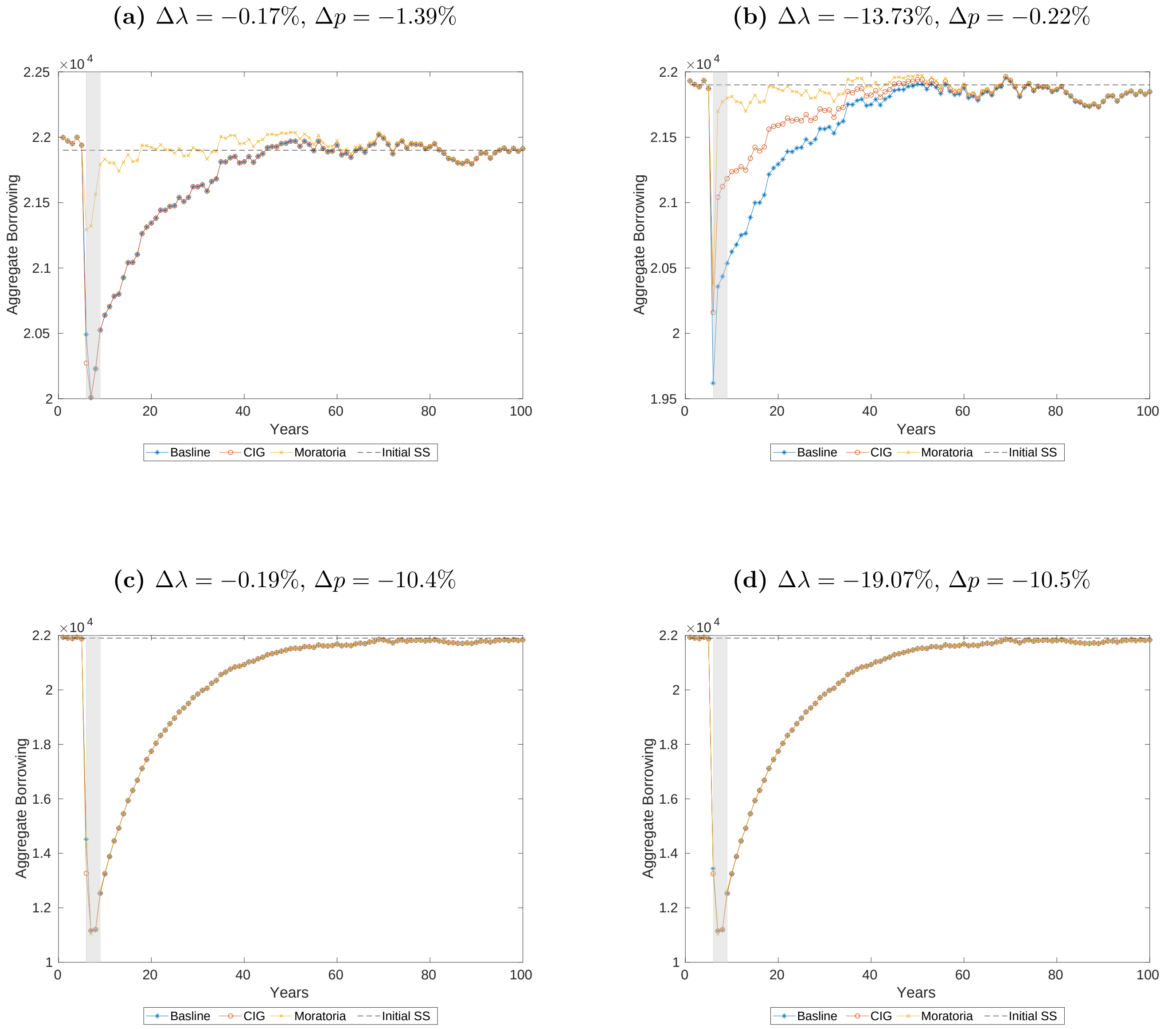
(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$

(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$

The shaded area represents the periods for which final good price p is below its original steady state value.

Corporate Finance

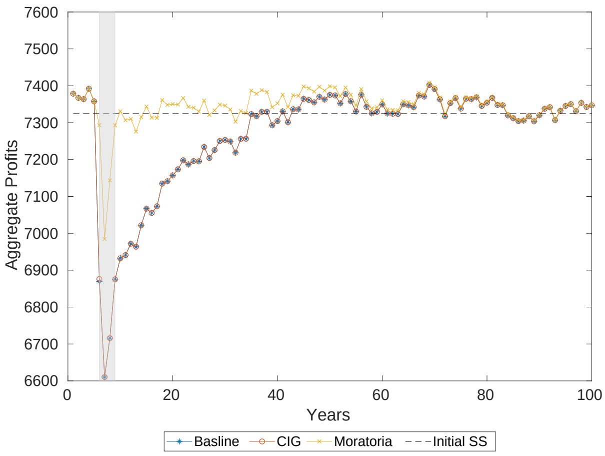
Figure 16: Aggregate Borrowing



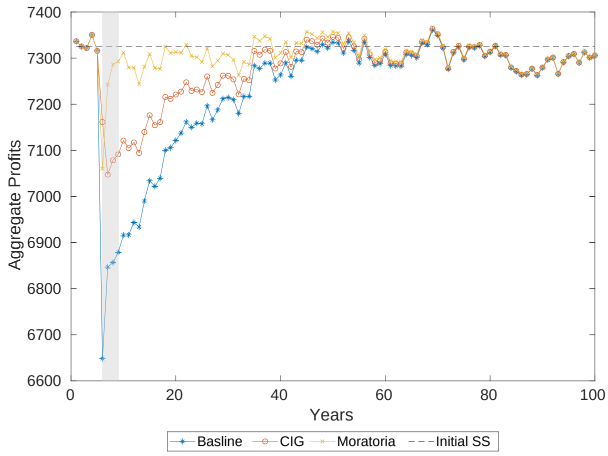
The shaded area represents the periods for which final good price p is below its original steady state value.

Figure 17: Aggregate Profits

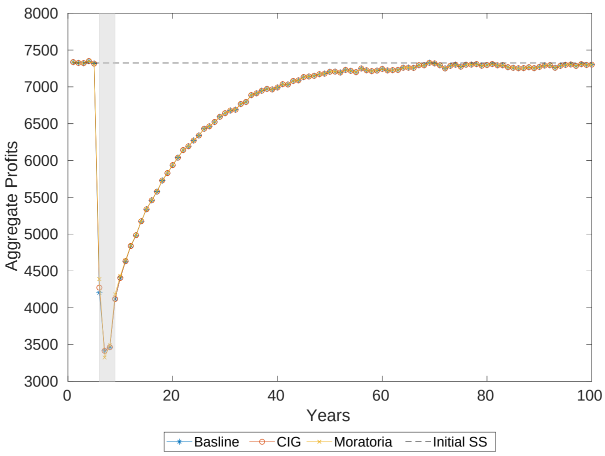
(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$



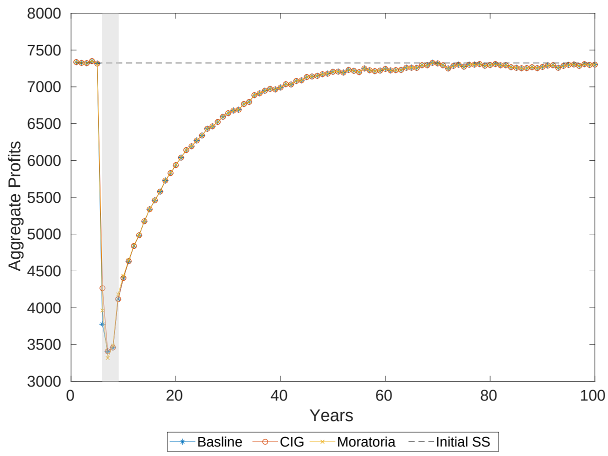
(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$



(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$



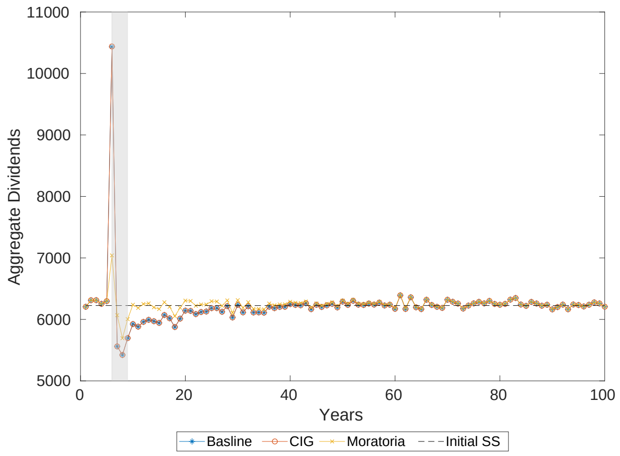
(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$



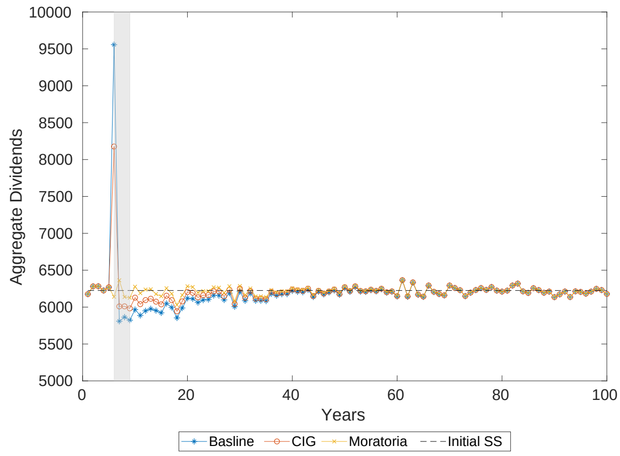
The shaded area represents the periods for which final good price p is below its original steady state value.

Figure 18: Aggregate Dividends

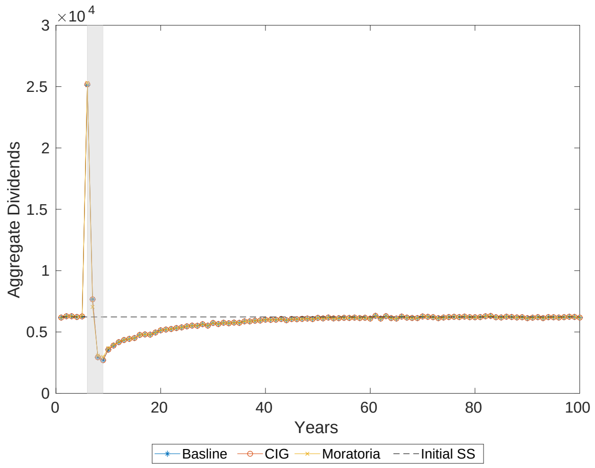
(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$



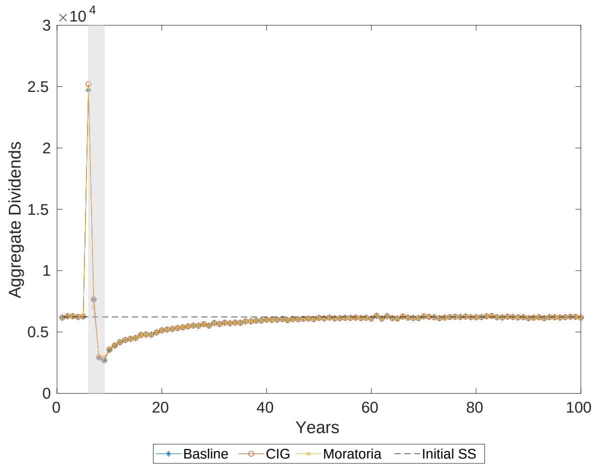
(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$



(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$



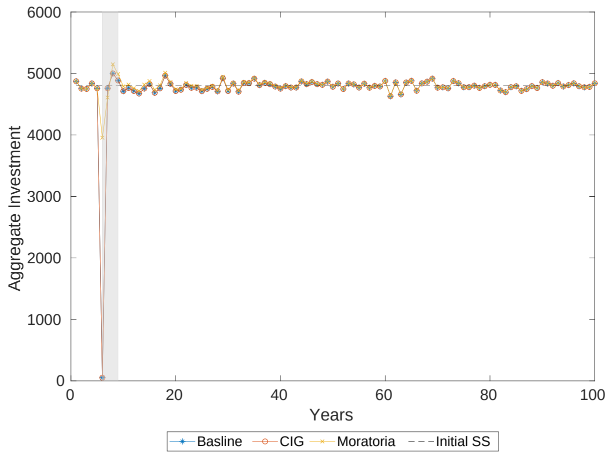
(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$



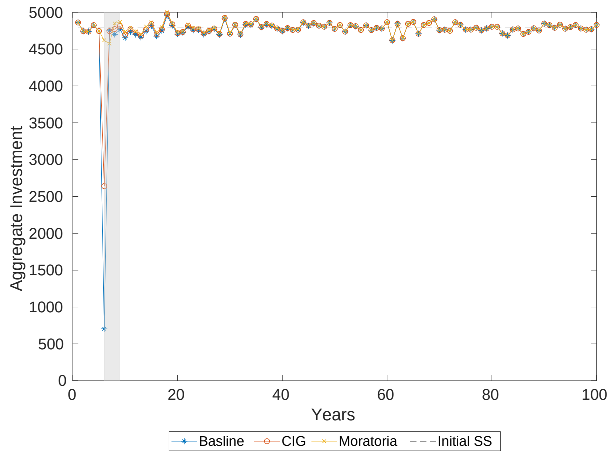
The shaded area represents the periods for which final good price p is below its original steady state value.

Figure 19: Aggregate Investment

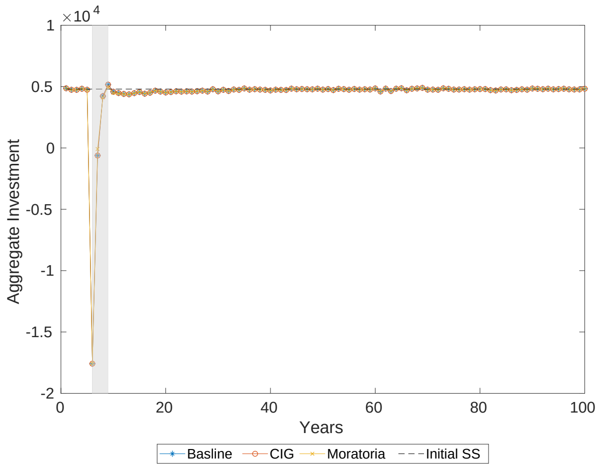
(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$



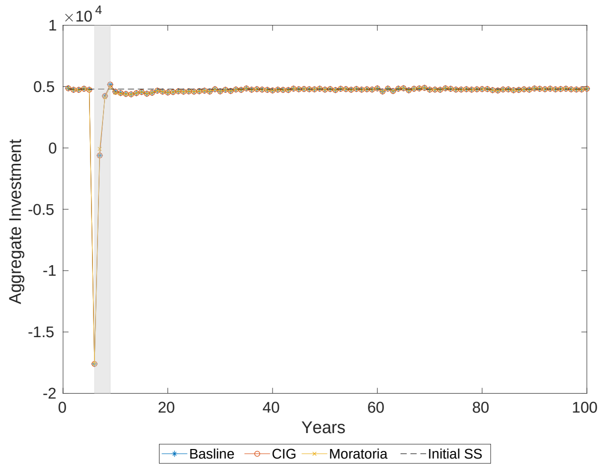
(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$



(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$



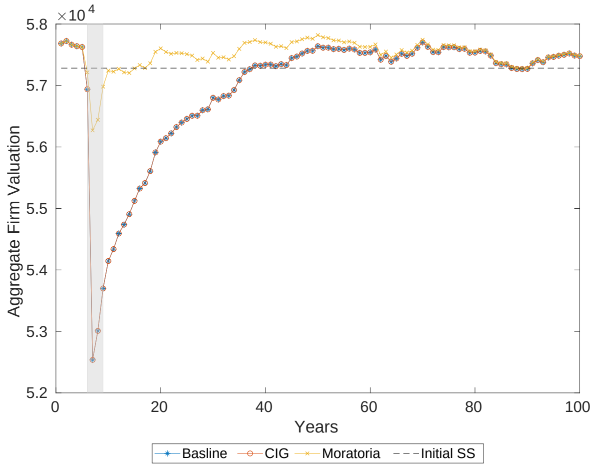
(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$



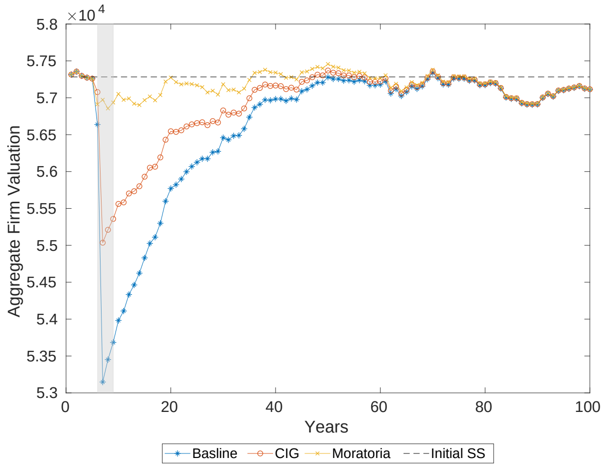
The shaded area represents the periods for which final good price p is below its original steady state value.

Figure 20: Aggregate Firm Valuation

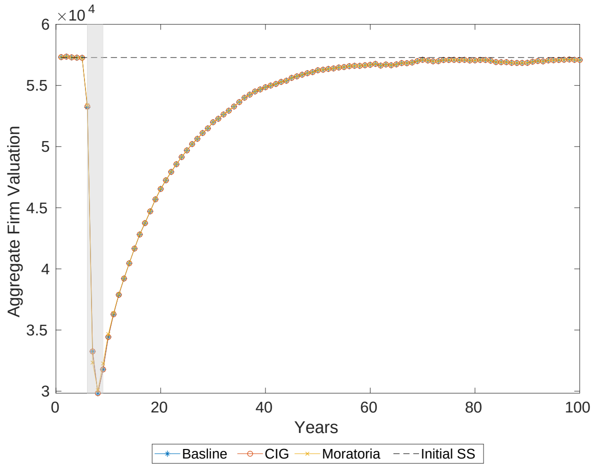
(a) $\Delta\lambda = -0.17\%$, $\Delta p = -1.39\%$



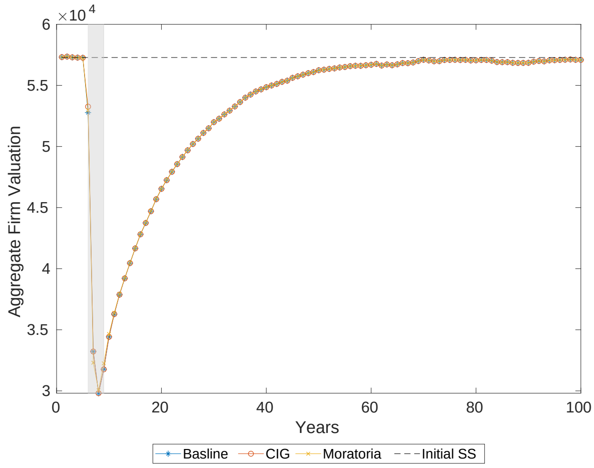
(b) $\Delta\lambda = -13.73\%$, $\Delta p = -0.22\%$



(c) $\Delta\lambda = -0.19\%$, $\Delta p = -10.4\%$



(d) $\Delta\lambda = -19.07\%$, $\Delta p = -10.5\%$



The shaded area represents the periods for which final good price p is below its original steady state value.

B Math Appendix

B.1 Permanent Productivity

Equation 18 shows cash flow in levels. Analogously to Midrigan and Xu (2014), assuming that $\alpha+\beta+\gamma=1$ and that fixed costs are proportional to permanent productivity, cash flow is homogeneous of degree 1 in (L, M, K, θ).

$$
\begin{aligned}
CF\left(L,M,K,\varepsilon,\theta\right) =& p\left(\theta\varepsilon\right)^{1-\nu}\left(L^{\alpha}K^{\beta}M^{\gamma}\right)^{\nu}-\left(wL+p_{m}M+\theta c_{f}\right)\left(1+r_{s}\right) \qquad (18)\\
=& p\theta\varepsilon^{1-\nu}\left(\frac{L^{\alpha}K^{\beta}M^{\gamma}}{\theta^{\alpha+\beta+\gamma}}\right)^{\nu}-\theta\left(w\frac{L}{\theta}+p_{m}\frac{M}{\theta}+c_{f}\right)\left(1+r_{s}\right)\\
=& p\theta\varepsilon^{1-\nu}\left(\left(\frac{L}{\theta}\right)^{\alpha}\left(\frac{M}{\theta}\right)^{\gamma}\left(\frac{K}{\theta}\right)^{\beta}\right)^{\nu}-\theta\left(w\frac{L}{\theta}+p_{m}\frac{M}{\theta}+c_{f}\right)\left(1+r_{s}\right)\\
CF\left(\lambda L,\lambda M,\lambda K,\varepsilon,\lambda\theta\right) =& p\lambda\theta\varepsilon^{1-\nu}\left(\left(\frac{\lambda L}{\lambda\theta}\right)^{\alpha}\left(\frac{\lambda M}{\lambda\theta}\right)^{\gamma}\left(\frac{\lambda K}{\lambda\theta}\right)^{\beta}\right)^{\nu}\\
&-\lambda\theta\left(w\frac{L}{\theta}+p_{m}\frac{M}{\theta}+c_{f}\right)\left(1+r_{s}\right) \qquad (19)\\
=& \lambda CF\left(L,M,K,\varepsilon,\theta\right)
\end{aligned}
$$

By setting $\lambda=\theta$, we arrive at Equation 1 in the text. This means the static problem of the firm is normalizable by its permanent TFP level. Given the liquidation cost function $c_l\left(K,K'\right)$ is homogeneous of degree 1 in (K,K'), it too is normalizable by θ. Therefore, all elements that compose the dynamic investment problem of firms are normalizable by permanent productivity, making the entire recursive problem scalable by θ.

B.2 Solution of the Static Problem

Maximizing Profits Firms choose how much labor l and intermediate input m to employ in production to maximize profits given in Equation 2. Note that either segment of $\pi\left(k,\varepsilon\right)$ is monotonically increasing in $CF\left(k,\varepsilon\right)-\tau_{w}wl\left(k,\varepsilon\right)$. Note also that $\pi\left(k,\varepsilon\right)$ is continuous and equal to δk at the threshold between both segments. Therefore, maximizing profits implies solving Equation 20.

$$
CF\left(k,\varepsilon\right)-\tau_{w}wl=\max_{l,m}p\varepsilon^{1-\nu}\left(l^{\alpha}k^{\beta}m^{\gamma}\right)^{\nu}-\left(wl+p_{m}m+c_{f}\right)\left(1+r_{s}\right)-\tau_{w}wl \qquad (20)
$$

The first order conditions are as follows for that problem state the following conditions:

$$l : l = \left(\frac{\alpha}{w\left(1 + r_s + \tau_w\right)} \nu p \varepsilon^{1-\nu} k^{\beta\nu} m^{\gamma\nu} \right)^{\frac{1}{1-\alpha\nu}}$$
$$m : m = \left(\frac{\gamma}{p_m\left(1 + r_s\right)} \nu p \varepsilon^{1-\nu} k^{\beta\nu} \right)^{\frac{1}{1-\gamma\nu}} l^{\frac{\alpha\nu}{1-\gamma\nu}}$$

Given these, we can solve for the level of inputs used as a function of TFP and capital:

$$l_\pi\left(k, \varepsilon\right) = \left(\nu p \varepsilon^{1-\nu} k^{\beta\nu} \left(\frac{\alpha}{w\left(1 + r_s + \tau_w\right)} \right)^{1-\gamma\nu} \left(\frac{\gamma}{p_m\left(1 + r_s\right)} \right)^{\gamma\nu} \right)^{\frac{1}{1-\nu(\gamma+\alpha)}}$$
$$m_\pi\left(k, \varepsilon\right) = \left(\nu p \varepsilon^{1-\nu} k^{\beta\nu} \left(\frac{\alpha}{w\left(1 + r_s + \tau_w\right)} \right)^{\alpha\nu} \left(\frac{\gamma}{p_m\left(1 + r_s\right)} \right)^{1-\alpha\nu} \right)^{\frac{1}{1-\nu(\alpha+\gamma)}}$$

Note that the ratio of the FOCs identify the ratio of output elasticities in the data:

$$\frac{\alpha}{\gamma} = \frac{wl\left(k, \varepsilon\right)}{p_m m\left(k, \varepsilon\right)} \frac{\left(1 + r_s + \tau_w\right)}{\left(1 + r_s\right)}$$

Finally, profit maximizing cash flow takes the following form:

$$CF_\pi\left(k, \varepsilon\right) = \left(\nu p \varepsilon^{1-\nu} k^{\beta\nu} \left(\frac{\alpha}{w\left(1 + r_s + \tau_w\right)} \right)^{\alpha\nu} \left(\frac{\gamma}{p_m\left(1 + r_s\right)} \right)^{\gamma\nu} \right)^{\frac{1}{1-\nu(\gamma+\alpha)}} \left(\nu^{-1} - \alpha \frac{1 + r_s}{1 + r_s + \tau_w} - \gamma \right) - c_f\left(1 + r_s\right)$$

Solvency Solvent firms are those who are able to produce positive cash-flow, irrespective of whether that maximizes their after-tax profits or not. Equation 21 represents the maximum amount of cash-flow firms can produce.

$$CF_m\left(k, \varepsilon\right) = \max_{l,m} p \varepsilon^{1-\nu} \left(l^\alpha k^\beta m^\gamma \right)^\nu - \left(wl + p_m m + c_f\right)\left(1 + r\right) \tag{21}$$

The first order conditions for labor and intermediates are as follows:

$$l : l = \left(\frac{\alpha}{w\left(1 + r_s\right)} \nu p \varepsilon^{1-\nu} k^{\beta\nu} m^{\gamma\nu} \right)^{\frac{1}{1-\alpha\nu}}$$
$$m : m = \left(\frac{\gamma}{p_m\left(1 + r_s\right)} \nu p \varepsilon^{1-\nu} k^{\beta\nu} \right)^{\frac{1}{1-\gamma\nu}} l^{\frac{\alpha\nu}{1-\gamma\nu}}$$

Given these, the level of cash flow maximizing inputs as a function of TFP and capital are:

$$l\left(k,\varepsilon\right)=\left(\nu p\varepsilon^{1-\nu}k^{\beta\nu}\left(\frac{\alpha}{w\left(1+r_{s}\right)}\right)^{1-\gamma\nu}\left(\frac{\gamma}{p_{m}\left(1+r_{s}\right)}\right)^{\gamma\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}}$$

$$m\left(k,\varepsilon\right)=\left(\nu p\varepsilon^{1-\nu}k^{\beta\nu}\left(\frac{\alpha}{w\left(1+r_{s}\right)}\right)^{\alpha\nu}\left(\frac{\gamma}{p_{m}\left(1+r_{s}\right)}\right)^{1-\alpha\nu}\right)^{\frac{1}{1-\nu(\alpha+\gamma)}}$$

And therefore, maximized cash flow takes the following form:

$$\begin{aligned}CF_{m}\left(k,\varepsilon\right)=&\left(\nu p\varepsilon^{1-\nu}k^{\beta\nu}\left(\frac{\alpha}{w\left(1+r_{s}\right)}\right)^{\alpha\nu}\left(\frac{\gamma}{p_{m}\left(1+r_{s}\right)}\right)^{\gamma\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}}\\&\left(\nu^{-1}-\alpha-\gamma\right)-c_{f}\left(1+r_{s}\right)\end{aligned}$$

Solving for ε and equalizing maximum cash flow to zero yields the bankruptcy threshold $\varepsilon_{B}\left(k\right)$ shown in Equation 8.

Firms with Negative Cash-flow when Maximizing Profits Because I assume decreasing returns to scale in all individual inputs, i.e. $\nu<1$ and $\alpha+\beta+\gamma=1$, cash flow under profit maximization is necessarily lower than the maximum cash flow firms can obtain, as shown in Equation 22.

$$CF_{m}\left(k,\varepsilon\right)-CF_{\pi}\left(k,\varepsilon\right)\propto\Lambda>0 \tag{22}$$

$$\Lambda=\left(\nu^{-1}-\gamma\right)-\frac{\alpha}{\left(1+r_{s}+\tau_{w}\right)}\left(\frac{\left(1+r_{s}+\tau_{w}\right)^{\frac{1-\gamma\nu}{1-\nu(\gamma+\alpha)}}-\left(1+r_{s}\right)^{\frac{1-\gamma\nu}{1-\nu(\gamma+\alpha)}}}{\left(1+r_{s}+\tau_{w}\right)^{\frac{\alpha\nu}{1-\nu(\gamma+\alpha)}}-\left(1+r_{s}\right)^{\frac{\alpha\nu}{1-\nu(\gamma+\alpha)}}}\right)$$

The choice of profit maximizing inputs can lead some firms to negative cash flow in spite of being capable of producing positive cash flow under a higher choice of labor. I thus assume that firms in that situation choose an amount of labor $l_{N}\left(k,\varepsilon\right)$ such that they produce just enough revenues to avoid going bankrupt, as illustrated in Equation 23.

$$\begin{aligned}CF_{N}\left(k,\varepsilon\right)-\tau_{w}wl=&\max_{l,m}p\varepsilon^{1-\nu}\left(l^{\alpha}k^{\beta}m^{\gamma}\right)^{\nu}-\left(wl+p_{m}m+c_{f}\right)\left(1+r_{s}\right)-\tau_{w}wl\\&s.t.p\varepsilon^{1-\nu}\left(l^{\alpha}k^{\beta}m^{\gamma}\right)^{\nu}\geq\left(wl+p_{m}m+c_{f}\right)\left(1+r_{s}\right)\\&l\geq0\end{aligned} \tag{23}$$

Defining $\{\lambda, \mu\}$ as the respective Lagrange multipliers on the solvency constraint and the non-negative labor constraint, the first order conditions of the problem yield:

$$\begin{aligned} l : l &= \left(\frac{(1-\lambda)\,\alpha\nu p\varepsilon^{1-\nu}\left(k^{\beta}m^{\gamma}\right)^{\nu}}{(1-\lambda)\,w\,(1+r_s)+\tau_w w+\mu}\right)^{\frac{1}{1-\alpha\nu}} \\ m : m &= \left(\frac{\gamma}{p_m\,(1+r_s)}\nu p\varepsilon^{1-\nu}k^{\beta\nu}\right)^{\frac{1}{1-\gamma\nu}} l^{\frac{\alpha\nu}{1-\gamma\nu}} \end{aligned}$$

Correspondingly, the choice of labor is implicitly defined by Equation 24 as follows:

$$l_N\,(k,\varepsilon) : \left(\frac{p\varepsilon^{1-\nu}k^{\beta\nu}}{(p_m\,(1+r_s))^{\gamma\nu}}\right)^{\frac{1}{1-\gamma\nu}} (\gamma\nu)^{\frac{\nu\gamma}{1-\gamma\nu}}\,(1-\gamma\nu)\,l^{\frac{\alpha\nu}{1-\gamma\nu}} - w\,(1+r_s)\,l = c_f\,(1+r_s) \quad (24)$$

Once firms evaluate the profits they obtain under this choice of labor, plus the value of staying in the market, they choose whether to go ahead and produce, or whether to rather liquidate the firm voluntarily, as represented by Equation 10.

B.3 Aggregate Productivity

$$\begin{aligned} L &= \int_i L_i di \\ K &= \int_i K_i di \\ M &= \int_i M_i di \end{aligned}$$

$$\begin{aligned} \frac{L_i}{L} &= \frac{\left((\theta_i\varepsilon_i)^{1-\nu}K_i^{\beta\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}}}{\int_i \left((\theta_i\varepsilon_i)^{1-\nu}K_i^{\beta\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}} di} \\ \frac{M_i}{M} &= \frac{\left((\theta_i\varepsilon_i)^{1-\nu}K_i^{\beta\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}}}{\int_i \left((\theta_i\varepsilon_i)^{1-\nu}K_i^{\beta\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}} di} \end{aligned}$$

$$
\begin{aligned}
Y_i\left(K_i,\varepsilon_i,\theta_i\right) &= \left(\theta_i\varepsilon_i\right)^{1-\nu}\left(L_i^{\alpha}K_i^{\beta}M_i^{\gamma}\right)^{\nu} \\
&= \left(\theta_i\varepsilon_i\right)^{1-\nu}\left(\frac{\left(\left(\theta_i\varepsilon_i\right)^{1-\nu}K_i^{\beta\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}}}{\int_i\left(\left(\theta_i\varepsilon_i\right)^{1-\nu}K_i^{\beta\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}}di}\right)^{\nu(\alpha+\gamma)}\left(L^{\alpha}K_i^{\beta}M^{\gamma}\right)^{\nu} \\
&= \left(\theta_i\varepsilon_i\right)^{1-\nu}\left(\frac{K_i}{K}\right)^{\beta\nu}\frac{\left(\left(\theta_i\varepsilon_i\right)^{1-\nu}K_i^{\beta\nu}\right)^{\frac{\nu(\alpha+\gamma)}{1-\nu(\gamma+\alpha)}}}{\left(\int_i\left(\left(\theta_i\varepsilon_i\right)^{1-\nu}K_i^{\beta\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}}di\right)^{\nu(\alpha+\gamma)}}\left(L^{\alpha}K^{\beta}M^{\gamma}\right)^{\nu}
\end{aligned}
$$

$$
Y = \frac{\int_i\left(\left(\theta_i\varepsilon_i\right)^{1-\nu}K_i^{\beta\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}}di}{K^{\beta\nu}\left(\int_i\left(\left(\theta_i\varepsilon_i\right)^{1-\nu}K_i^{\beta\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}}di\right)^{\nu(\alpha+\gamma)}}\left(L^{\alpha}K^{\beta}M^{\gamma}\right)^{\nu}
$$

Assuming that $\int_i \theta_i di = 1$, given that $\alpha+\beta+\gamma=1$, and because transitory TFP and all normalized input choices are independent of θ_i, aggregate TFP amounts to:

$$
\begin{aligned}
TFP &= \left(\int_i \theta_i k_i di\right)^{-\beta\nu}\left(\int_i\left(\theta_i^{1-\nu(1-\beta)}\varepsilon_i^{1-\nu}k_i^{\beta\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}}di\right)^{1-\nu(\alpha+\gamma)} \\
&= \left(\int_i \theta_i di\int_i k_i di\right)^{-\beta\nu}\left(\int_i \theta_i^{\frac{1-\nu(1-\beta)}{1-\nu(\gamma+\alpha)}}di\int_i\left(\varepsilon_i^{1-\nu}k_i^{\beta\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}}di\right)^{1-\nu(\alpha+\gamma)} \\
&= \left(\int_i k_i di\right)^{-\beta\nu}\left(\int_i\left(\varepsilon_i^{1-\nu}k_i^{\beta\nu}\right)^{\frac{1}{1-\nu(\gamma+\alpha)}}di\right)^{1-\nu(\alpha+\gamma)}
\end{aligned}
$$

B.4 Misallocation

In an efficient allocation of inputs, the marginal productivity of each input should be equal across firms. Using the solution to Equation 2 computed in Appendix B.2, I compute the marginal productivity of each firm as follows:

$$
\begin{aligned}
MRPL\left(k,\varepsilon\right) &= \alpha\nu p\varepsilon^{1-\nu}\left(k^{\beta}m\left(k,\varepsilon\right)^{\gamma}\right)^{\nu}l\left(k,\varepsilon\right)^{\alpha\nu-1} = w\left(1+r_s+\tau_w\right) \\
MRPM\left(k,\varepsilon\right) &= \gamma\nu p\varepsilon^{1-\nu}\left(l\left(k,\varepsilon\right)^{\alpha}k^{\beta}\right)^{\nu}m\left(k,\varepsilon\right)^{\gamma\nu-1} = p_m\left(1+r_s\right)
\end{aligned}
$$

This shows that the static choices are efficient: firms have full information and are completely unconstrained when choosing their flexible inputs. Their choices of capital on the other hand are constrained by the fact that they do not know the productivity under which they will operate in the following period:

$$\begin{aligned} MRPK\left(k,\varepsilon\right) =&\beta\nu p\varepsilon^{1-\nu}\left(l\left(k,\varepsilon\right)^{\alpha}m\left(k,\varepsilon\right)^{\gamma}\right)^{\nu}k^{\beta\nu-1} \\ =&\beta\left(\nu p\left(\frac{\alpha}{w\left(1+r_s+\tau_w\right)}\right)^{\alpha\nu}\left(\frac{\gamma}{p_m\left(1+r_s\right)}\right)^{\gamma\nu}\left(\frac{\varepsilon}{k}\right)^{1-\nu}\right)^{\frac{1}{1-\nu(\alpha+\gamma)}} \end{aligned}$$

In order for MRPK to be equalized across firms, it has to be that firms with higher TFP have higher capital installed. If they could re-optimize their capital stock, the market would be able to equalize MRPK across firms as follows:[23]

$$\begin{aligned} &\max_{k^e}\varepsilon^{\frac{1-\nu}{1-\nu(\alpha+\gamma)}}\left(p\nu\left(k+k^e\right)^{\beta\nu}\left(\frac{\alpha}{w\left(1+r_s+\tau_w\right)}\right)^{\alpha\nu}\left(\frac{\gamma}{p_m\left(1+r_s\right)}\right)^{\gamma\nu}\right)^{\frac{1}{1-\nu(\alpha+\gamma)}} \\ &\left(\nu^{-1}-\alpha\frac{1+r_s}{1+r_s+\tau_w}-\gamma\right)-\left(r_s+\delta\right)k^e \end{aligned}$$

With the corresponding first order conditions:

$$\begin{aligned} \left(k+k^e\right) =&\varepsilon\left(\frac{\nu}{1-\nu\left(\alpha+\gamma\right)}\right)^{\frac{1}{1-\nu(\alpha+\gamma)}}\left(\nu^{-1}-\alpha\frac{1+r_s}{1+r_s+\tau_w}-\gamma\right)^{\frac{1-\nu(\alpha+\gamma)}{1-\nu}} \\ &\left(p\nu\left(\frac{\alpha}{w\left(1+r_s+\tau_w\right)}\right)^{\alpha\nu}\left(\frac{\beta}{\left(r_s+\delta\right)}\right)^{1-\nu(\alpha+\gamma)}\left(\frac{\gamma}{p_m\left(1+r_s\right)}\right)^{\gamma\nu}\right)^{\frac{1}{1-\nu}} \end{aligned}$$

Therefore, I obtain that the efficient MRPK takes the following constant form:

$$MRPK\left(k,\varepsilon\right)=\left(r_s+\delta\right)\frac{1-\nu\left(\alpha+\gamma\right)}{\left(1-\alpha\nu\frac{1+r_s}{1+r_s+\tau_w}-\gamma\nu\right)}$$

C Solution Method

C.1 Ergodic Steady State

I solve for the ergodic steady state equilibrium of the economy as follows:

1. Find numerical approximations for $V_i\left(k,\varepsilon;p\right)$, $k'\left(k,\varepsilon;p\right)$, $\xi_e\left(k,\varepsilon;p\right)$, $\xi_l\left(k,\varepsilon;p\right)$, and $\xi_B\left(k,\varepsilon;p\right)$ using value function iteration over equally-spaced grids for k and ε, and a given price p:

[23] Assuming they could borrow the corresponding funds at rate r.

(a) Provide a guess for the value of investing V_i^j.

(b) Obtain an updated guess V_i^{j+1} by solving Equation 4, which implies:

i. Solving for maximized cash flow $CF_m(k, \varepsilon)$ and obtaining the bankruptcy function $\xi_B(k, \varepsilon; p)$ as in Equation 7.

ii. Solving for the profit function $\pi_m(k, \varepsilon)$ in Equation 3.

iii. Solving for the liquidation function $\xi_l(k, \varepsilon; p)$ in Equation 10.

iv. Solving for the exit function $\xi_e(k, \varepsilon; p)$ in Equation 12.

v. Solving for the expected continuation value, given by Equation 13.

(c) Compare the initial guess V_i^j and V_i^{j+1}. If close enough, use the latter as the numerical approximation of $V_i(k, \varepsilon; p)$ and store the corresponding solution to Equation 13 as the numerical approximation of $k'(k, \varepsilon; p)$ as well as the bankruptcy, liquidation, and exit functions computed above; otherwise, go back to step 1a using V_i^{j+1} as an initial guess.

2. Given an approximation for $V_i(k, \varepsilon; p)$, and the expected continuation value computed in step 1(b)v, solve for the amount of capital potential entrants invest in, $k_e(p)$, as illustrated in Equation 6.

3. Given the approximations to the policy functions for investment of incumbents $k'(k, \varepsilon; p)$ and potential entrants $k_e(p)$, as well as the bankruptcy $\xi_B(k, \varepsilon; p)$, liquidation $\xi_l(k, \varepsilon; p)$, and exit functions $\xi_e(k, \varepsilon; p)$, obtain an approximation of the ergodic distribution of firms $\Phi(k, \varepsilon)$ on equally-spaced grids for k and ε:

(a) Provide a guess Φ^j for the distribution over the state space.

(b) Use Equation 14 and the policy functions computed above to obtain the following iteration for the state space distribution Φ^{j+1}.

(c) Compare Φ^j and Φ^{j+1}. If close enough, use the latter as the approximation to the steady state distribution of firms; otherwise, go back to step 3a using Φ^{j+1} as the new guess.

C.2 Simulation of the COVID-19 Shock

The simulation of the COVID-19 shock requires simulating the behavior of firms and the entire economy outside the ergodic steady state equilibrium as follows:

1. Set an exogenous path for the final good price $\{p_t\}_{t=0}^{\infty}$.

2. Solve for the value- and policy functions $V_i\left(k,\varepsilon;p\right)$, $k'\left(k,\varepsilon;p\right)$, $\xi_e\left(k,\varepsilon;p\right)$, $\xi_l\left(k,\varepsilon;p\right)$, and $\xi_B\left(k,\varepsilon;p\right)$ under the initial and final prices, as detailed in steps 1a through 1c in Section C.1.

3. Given the value- and policy functions under the final price level, compute the value- and policy functions under a forward-looking equilibrium of the economy as defined in Section 2 for the prices between the final and first levels by backward induction.

4. Given the initial price for the final good, and the corresponding policy functions, simulate a panel of firms a long enough number of periods so as it replicate the ergodic steady state distribution computed in steps 3a through 3c in the previous section.

5. Starting from the simulated initial steady state distribution, simulate the choices of firms and the trajectory of the economy once the price of the final good changes, using the policy functions computed in step 2 and the transition function described by Equation 14.