A theory of cyclic bribery

Maurizio Lisciandra, Antonio Miralles and Fabio Monteforte

Abstract

We develop a model in which a briber decides on an optimal schedule of bribing to the governing party in a bipartisan system. Detected corruption increases voters' resentment, while periods without corruption lowers resentment. Resentment increases the risk of political overturn, hence rising the minimum acceptable bribe. The main assumption is that the briber's discount rate is higher than the rate at which resentment diminishes when there is no corruption. The optimal schedule describes cycles alternating periods of bribing with periods of no corruption iff there is a resentment level at which bribing gives no positive instant gain to the briber.

Key words: Corruption, cyclicity, bribes.

1 Introduction

We present a theoretical model with the purpose of characterizing dynamic patterns of corruption. The model incorporates standard elements in political economy such as possible turnovers in a bipartisan system, and a briber who captures the ruling party in search for a rent. Voters' attitude is summarized by their resentment against repeated uncovering of corruption events.

Resentment grows or decreases depending on the amount of detected corruption cases. Resentment increases the peril of electoral turnover, making corrupting the governing party more expensive for a briber.

In such setup, we identify a clear-cut pattern in the dynamics of corruption. If there is a level of resentment that makes bribing not worthwhile with respect to the briber's instant payoff, then the optimal pattern of corruption describes repeated cycles of corruption activity and inactivity. If bribing generates instead positive instant returns to the briber regardless the level of resentment in the society, then the optimal bribing schedule describes no cycles. In this latter case, corruption activity initializes at some optimal moment in time and then it never stops.

The calculation of the optimal bribe, whenever the briber decides to be active, is based on the model by Shapiro and Stiglitz (1984) for the labor market. In that paper, there are two states for the worker: employed and unemployed. There are also two strategies: shirking and paying the required effort. Analogously, our model has two states for a party: in charge or in opposition. There are two strategies: accepting bribes and not accepting them. The briber plays the role the firm does in Shapiro and Stiglitz, with the difference that, in our model, the briber faces no competion. Or, in other words, the briber acts as a principal in a mechanism, allowing her to capture the ruling party in exchange of its minimum acceptable bribe.

The paper is structured as follows. The remaining parts of this Section 1 analyze evidence and related literature. Section 2 introduces de model. Section 3 solves for the optimal bribe when the briber is active. Section 4 solves for the optimal bribing schedule. Section 5 summarizes and discusses a possible future extension. An Appendix presents proofs of instrumental results used in the main derivations.

Literature

Andersson (2003) estimates that the impact of corruption on political support towards the government is negative and significant, hence validating one of the key state variables in our model, namely anger,or animosity against the ruling party. Our model, for the sake of being parsimonious, does not include the subtleties of partisanship, which shades the intensity of such negative impact on political support.

Bicchieri and Duffy (1997) develop an interesting theory of corruption cyclicity. The idea is as follows. The governing party has amassed a certain amount of resouces from a previous period of honest management. At some point, it is optimal to start illegally assigning those resources, if the benefitted organizations are able to provide votes to the ruling party, which the party needs to ensure reelection. When resources are depleted, this mechanism crumbles, giving rise to a new period of honesty.

In a similar fashion, Feitchinger and Wirl (1994) construct a model in which a politician must find her optimal intertemporal path, when her utility depends on 1) the gains obtained through corruption, and 2) the reputational loss due to corruption uncoveries. Cyclic variations in the intensity of corruption persist, with unstable cycles arising when the second element in the politician's utility function has low enough importance. This is, to our knowledge, the theoretical model that most resembles ours.

There are two elements we wish to point out here. Ours is a model in which bribers represent the elite who is able to set the rules of the mechanism. Political parties are passive agents who simply accept of reject bribes. This active role for bribers is a distinctive element of our model in the literature on corruption cycles. We regard our model as complementary to previous literature. Even in (democratic) societies where politicians are easily captured by bribers, corruption cycles become a possible outcome.

However, the second point we rise here is that, even with these similarities, testable predictions differ. In Feitchinger and Wirl, sufficiently high tolerance towards corruption (i.e. lower reputational effects) yield unstable cycles. Our model, in contrast, predicts no cycles in such more extreme cases.

Dawid and Feichtinger (1996), analyze an optimal path of corruption intensity for a bureaucrat that is affected by reputational loss. Interestingly, non-repelling paths point towards extreme limits, where corruption is either maximal or nonexistant. Although the paper focus on the limit level of corruption, there are interesting dynamics in the the optimal path. In a linear utility setup, corruption intensity changes are abrupt, hence alternating from maximal to minimal corruption regimes.

Rinaldi, Feichtinger and Wirl (1998) explore a more flexible dynamic model not involving the calucaltion of an optimal path. They match the parameters of the model in order to fit the history of corruption in Italy since 1948. States variables are a) support to politicians (popularity/reputation), b) hidden assets used in corrupted activies, c) investigation effort. The model might yield a "weak control system", describing an infinite cycle that follows the following stages: 1) increase of popularity with low corruption and low investigation effort, 2) efficient government, 3) rise of corruption and cumulation of hidden assets, 4) decline of popularity and increase in investigation efforts, 5) depletion of assets, less corruption and decline in investigation efforts, and 6) stagnation (low values for all state variables.)

Corruption has also been modelled as the cooperation/defection dilemma we find in games such as the paradigmatic prisoner's dilemma. Recently, Lee et al. (2019) take an evolutionary game theory approach with several strategic traits combining altruism/egoism with optimism/prudency. Computations result in a cycle of honest and corrupt umpires.

Lastly, there is a subset of literature devoted to the connection between corruption and the political election cycle. Intuitively, following the idea of democratic accountability, the months before important elections should be characterized by a more prudent behavior, with less corruption. Nevertheless, this argument is not so clear. Bribers would also want to commit the politicians immediately before the election takes place, while politicians may need funds for their electoral campaigns to be effective. Figueroa (2020), for the case of an uncovered corruption network in Argentina, finds bribes to be significantly higher two weeks before- than two weeks after the election. Cooper (2021) finds similar results for the case of African elections in democratic regimes.

2 The model

We construct a continuous time model. In each moment $t \geq 0$, a briber has an instant rent r to make though an illegal favorable decision from the governing party. The briber decides whether to bribe, and by how much, the party in office at time t, in order to obtain the rent r. We denote such bribe with $b(t)$. The ruling party simply decides, whenever a bribe is offered, if it accepts the bribe or not.

When bribing is undetaken, there is an instant probability π it is detected by judiciary authorities. Such detection is rendered public to the ruled citizens. Since time is continuous, a time span of corruption activity of lenght 1 will be detected in a portion π and remain undetected in a portion $1 - \pi$. Detection implies a penalty for both the ruling party and the briber: not only no bribe or rent is perceived but both agents must pay a penalty equal to a proportion s of the intended rents (moreover, the paid bribe is a sunk cost for the briber.)

Citizens summarize their sentiments about the political system through a resentment variable $\lambda(t)$, with $\lambda(0) = \lambda_0 > 0$. The dynamics of $\lambda(t)$ are as follows: $\frac{d\log\lambda(t)}{dt} = \omega > 0$ when corruption is detected at time t, $\frac{d\log\lambda(t)}{dt} = -\phi < 0$ when corruption is not detected. Let $\varphi = \pi\omega - (1-\pi)\phi$ denote the expected increase rate of resentment when the ruling party is corrupt at time t.

The political system is bipartisan. There is a probability per unit of time that the governing party is overturned by the competing party. Such probability per unit of time is the sum of two elements: $\theta > 0$ which is exogenous, and, only in the event corruption has been detected, $\lambda(t)$. Being in office gives the ruling party an instant payoff $m > 0$. Only the governing party can receive bribes.

Political parties discount future payoffs at rate $\rho > 0$. The briber discounts future payoffs at rate $\tilde{\rho} > 0$.

We assume that $\tilde{\rho} \geq \phi$. This is the main assumption of the model to make it tractable. This assumption means that the briber discounts future payoffs at a higher rate than the one at which voters could decrease their resentment. A brief discussion on such assumption is included in Section 5.

We solve for the optimal bribing schedule from the point of view of the briber, who aims to maximize the present value of her flow of payoffs on an infinite horizon.

3 Minimum acceptable instant bribes

In this section, for the sake of notational simplicity, we suppress the notation regarding time.

A ruling party that does not accept a bribe b obtains an instant payoff m. If the bribe is accepted, her instant payoff becomes $\tilde{m} = m + (1-\pi)b - \pi sr$. The briber can commit to not offering any more bribes in the future if one bribe is rejected.

Let X and Y be the ruling and the competing party at time t, respectively. The subscript c (for corrupt) denotes a ruling party that is willing to accept a bribe when in charge, whereas the subscript h (for honest) denotes the willingness to reject any bribe offer.

We follow the derivations of Shapiro and Stiglitz (1984.) For a type h, the instant probability of turnover is constant in time, since it is just θ. For this reason, the present value of (present and future) expected utility is constant in time for both parties X and Y. Considering an arbitrarily small time lapse T, party X faces the following characterization of such present value:

$$EU_{X_h} = mT + e^{-\rho T}\left[\theta T \cdot EU_{Y_h} + (1-\theta T)EU_{X_h}\right]$$

Using an approximation $e^{-\rho T} \approx 1 - \rho T$, and taking the limit when $T \to 0$, we obtain the so-called "asset equation"

$$\rho EU_{X_h} = m - \theta[EU_{X_h} - EU_{Y_h}]$$

Similarly, we obtain the asset equation of party Y when being of type h:

$$\rho EU_{Y_h} = \theta[EU_{X_h} - EU_{Y_h}]$$

Solving this simple system, we obtain

$$EU_{X_h} = \frac{\rho + \theta}{\rho[\rho + 2\theta]} m$$

$$EU_{Y_h} = \frac{\theta}{\rho[\rho + 2\theta]} m$$

Now, when parties engage into corruption, the probability of turnover $\theta + \lambda\pi$ is time-varying since λ is so. We cannot take for granted that the present value of expected utilities is constant over time. However, the briber pays the minimum possible bribe, the one that keeps the ruling party indifferent between accepting it or declining it. And, by declining the bribe, the present value of expected payoffs remains constant in time. Hence, in our model, both EU_{X_c} and EU_{Y_c} are constant in time.

Present values of expected utilities are then characterized by the following asset equations:

$$\begin{aligned} \rho EU_{X_c} &= \tilde{m} - (\theta + \lambda\pi)[EU_{X_c} - EU_{Y_c}] \\ \rho EU_{Y_c} &= (\theta + \lambda\pi)[EU_{X_c} - EU_{Y_c}] \end{aligned}$$

The system of equations yields solution

$$EU_{X_c} = \frac{\rho + \theta + \lambda\pi}{\rho[\rho + 2(\theta + \lambda\pi)]} \tilde{m}$$

$$EU_{Y_c} = \frac{\theta + \lambda\pi}{\rho[\rho + 2(\theta + \lambda\pi)]} \tilde{m}$$

We now make use of the indifference condition, $EU_{X_c} = EU_{X_h}$, or

$$\frac{\tilde{m}}{m} = \frac{(\rho + \theta)(\rho + 2\theta) + 2\lambda\pi(\rho + \theta)}{(\rho + \theta)(\rho + 2\theta) + \lambda\pi(\rho + 2\theta)}$$

Given that $\frac{\tilde{m}}{m} = 1 + (1 - \pi)\frac{b}{m} - \pi\frac{sr}{m}$, the optimal bribe (conditional on bribing) is

$$b = \frac{\pi}{1 - \pi}\left[sr + \frac{\lambda\rho m}{(\rho + \theta)(\rho + 2\theta) + \lambda\pi(\rho + 2\theta)}\right]$$

Notice that b is increasing in λ. This leaves an instant payoff for the briber of the following form:

$$\gamma(\lambda) = (1 - \pi)r - \pi sr - \frac{\pi}{1 - \pi}\left[sr + \frac{\lambda\rho m}{(\rho + \theta)(\rho + 2\theta) + \lambda\pi(\rho + 2\theta)}\right]$$

4 The optimal schedule of bribing

Based on the previous section, we make the following notational shortenings:

$$
\begin{aligned}
A &= (1-\pi)r - \pi sr - \frac{\pi}{1-\pi}sr \\
B &= \frac{\pi\rho m}{1-\pi} \\
a &= (\rho+\theta)(\rho+2\theta) \\
b &= \pi(\rho+2\theta)
\end{aligned}
$$

This notation allows us to write the briber's instant payoff as a funtion of λ

$$\gamma(\lambda) = A - B\frac{\lambda}{a+b\lambda}$$

We model the dynamic problem for the briber, thus we retake the notation for time when necessary from now on. The briber designs her schedule of bribing activity in order to maximize the following present value of present and future payoffs:

$$\max_{1_{Bribe}} \mathcal{P} = \int_0^\infty e^{-\tilde{\rho}t} 1_{Bribe}(t)\gamma(\lambda(t))dt$$

subject to $\lambda(0) = \lambda_0$, $\frac{\dot{\lambda}}{\lambda} = \varphi$ if $1_{Bribe}(t) = 1$, $\frac{\dot{\lambda}}{\lambda} = -\phi$ if $1_{Bribe}(t) = 0$. Obviously, $1_{Bribe}(t)$ takes value 1 when the briber decides to bribe, and value 0 when the briber remains inactive instead.

This dynamic programming model is solved assuming that there is a stationary solution consisting of the repetition of activity-inactivity cycles. Thus the solution involves a first stage in which depending on the initial resentment state λ_0 we decide the resentment state λ^* at which we initiate repetitive corruption activity-inactivity cycles. In a second stage, depending on λ^*, we select the optimal lenght T^* of corruption activity in each repeated cycle. At the end, we check that the choice of λ^* does not depend on λ_0, validating the stationary solution approach.

The following lemma will be most useful when solving the problem.

Lemma 1: Let t be a moment at which corruption activity ceases and t' its closest next moment at which the briber retakes corruption activity. If the briber follows an optimal schedule, we must have

$$e^{-\tilde{\rho}t}\gamma(\lambda(t)) = e^{-\tilde{\rho}t'}\gamma(\lambda(t'))$$

Proof: Suppose $e^{-\tilde{\rho}t}\gamma(\lambda(t)) < e^{-\tilde{\rho}t'}\gamma(\lambda(t'))$. Consider an alternative schedule that is identical to what the briber is already undertaking except for the following change: instead of ceasing activity at t, she ceases activity at $t-\mu$, and instead of retaking it at t' she retakes activity at $t'-\mu$. μ is a very small positive number.

Notice that $\log\lambda(\cdot)$ (and therefore the briber's instant payoff) is unchanged in the time intervals $[0, t-\mu]$ and $[t', \infty)$. Therefore, for μ small enough,

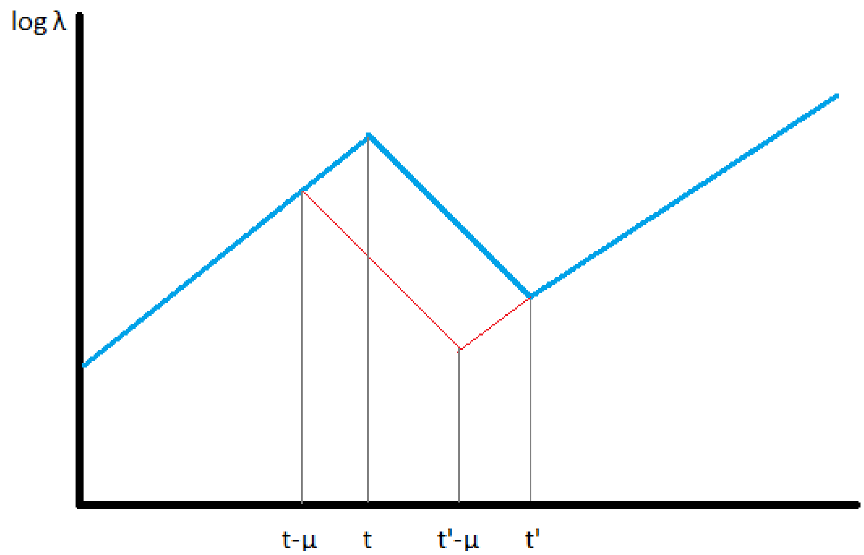

Figure 1: Illustrating the proof of Lemma 1. By stopping activity a bit earlier, the briber substitutes periods with lower discounted payoff with periods with higher discounted payoffs.

$e^{-\tilde{\rho}t}\gamma(\lambda(t)) < e^{-\tilde{\rho}t'}\gamma(\lambda(t'))$ implies that the suggested modification improves briber's overall payoff.

Suppose $e^{-\tilde{\rho}t}\gamma(\lambda(t)) > e^{-\tilde{\rho}t'}\gamma(\lambda(t'))$. The payoff-improving modification would consist of enlargening the activity interval to $t + \mu$, retaking corruption activity at $t' + \mu$. We skip a more detailed argument since it would be a repetition of the previous one. **QED**

The assumption $\tilde{\rho} \geq \phi$ ensures that a unique solution (if any) exists for the lenght of each cycle. The assumption $A < B/b$ guarantees that a solution exists. All this is shown through Lemma A1 at the Appendix.

We start with the solution for the optimal bribing activity time lenght T^* conditional on λ^*, the (optimally chosen) level of resentment at which corruption activity restarts. Notice that the total lenght of each repeated activity-inactivity cycle equals $\frac{\varphi+\phi}{\phi}T^*$. Since $\log \lambda$ is piece-wise linear in t, it takes $\frac{\varphi}{\phi}T^*$ units of time to go back to the initial resentment state, after a time lenght T^* of bribing activity.

Let $\Psi \equiv \tilde{\rho}\frac{\varphi+\phi}{\phi}$. Fixing λ^*, T^* maximizes the following function:

$$\begin{aligned} O(T;\lambda^*) &\equiv \sum_{n=0}^{\infty}\int_{n\frac{\varphi+\phi}{\phi}T}^{n\frac{\varphi+\phi}{\phi}T+T} e^{-\tilde{\rho}t}\gamma(\lambda^* e^{\varphi t})dt \\ &= \frac{1}{1-e^{-\Psi T}}\int_0^T e^{-\tilde{\rho}t}\gamma(\lambda^* e^{\varphi t})dt \\ &\equiv \frac{I(T;\lambda^*)}{1-e^{-\Psi T}} \end{aligned}$$

where $I(T;\lambda^*)$ denotes the former integral. The first order condition on T yields

$$\begin{aligned} O_T(T^*;\lambda^*) &= \frac{I_T(T^*;\lambda^*)}{1-e^{-\Psi T^*}} - \Psi e^{-\Psi T^*}\frac{I(T^*;\lambda^*)}{(1-e^{-\Psi T^*})^2} \\ &= \frac{1}{1-e^{-\Psi T^*}}\left[I_T(T^*;\lambda^*) - \Psi e^{-\Psi T^*} O(T^*;\lambda^*)\right] = 0 \end{aligned}$$

We now apply Lemma 1, which adapted to the current maximization problem can be written as[1]

$$I_T(T^*;\lambda^*) \equiv e^{-\tilde{\rho}T^*}\gamma(\lambda^* e^{\varphi T^*}) = e^{-\Psi T^*}\gamma(\lambda^*)$$

This observation yields the simple

$$O_T(T^*;\lambda^*) = \frac{\gamma(\lambda^*)}{\Psi}$$

Actually, we do not need to know the exact choice of T^* in order to obtain the briber's optimal payoff. We now study the first stage, that is, the calculation of the optimal λ^*.

We assume that λ_0 is sufficiently high so that the briber optimally remains inactive until resentment lowers down to λ^*. Otherwise the optimal schedule would consist of being active until some time t' at which the resentment state takes some value λ_1 and then solve for the problem below (yet using λ_1 instead of λ_0.) Since the optimal λ^* (and also T^*) is found to be invariant with respect to λ_0 in the problem below, it is also invariant to the choice of λ_1 (and associated t'.) The briber's overall payoff would be sensitive to this initial period of activity yet the stationary part of the solution would not.

We choose λ^* to maximize

$$M(\lambda;\lambda_0) \equiv e^{-\tilde{\rho}\tau(\lambda;\lambda_0)}\frac{\gamma(\lambda)}{\Psi}$$

[1] A more rigorous approach would include this equation as a constraint in the maximization program and then it would construct a Lagrangian. Since this constraint is a condition for maximization and not an exogenous constraint, the value of its associated Lagrange multiplier would be zero, as the constraint does not interfere with the maximum attainable value of the objective function. The solution to this problem would be equivalent to that presented in the main text.

with $\tau(\lambda;\lambda_0)$ defined by

$$\lambda \equiv \lambda_0 e^{-\phi\tau(\lambda;\lambda_0)}$$

or

$$e^{-\tilde{\rho}\tau(\lambda;\lambda_0)} = \left(\frac{\lambda}{\lambda_0}\right)^{\frac{\tilde{\rho}}{\phi}}$$

thus

$$M(\lambda;\lambda_0) = \left(\frac{\lambda}{\lambda_0}\right)^{\frac{\tilde{\rho}}{\phi}} \frac{\gamma(\lambda)}{\Psi}$$

The first order condition is then

$$\tilde{\rho}\left(A - B\frac{\lambda^*}{a+b\lambda^*}\right) - aB\phi\frac{\lambda^*}{(a+b\lambda^*)^2} = 0$$

Proposition 1: If $\tilde{\rho} \geq \phi$ and $A < B/b$, then λ^* exists, it is unique, and it yields a maximum for $M(\lambda;\lambda_0)$.

Proof: It stems from the proof of Lemma A1 in the Appendix.

$$\begin{aligned} M_\lambda(\lambda;\lambda_0) &= \left(\frac{\lambda}{\lambda_0}\right)^{\frac{\tilde{\rho}}{\phi}} \frac{1}{\Psi\lambda\phi} \cdot \\ &\cdot \left[\tilde{\rho}\left(A - B\frac{\lambda}{a+b\lambda}\right) - aB\phi\frac{\lambda}{(a+b\lambda)^2}\right] \end{aligned}$$

If $\tilde{\rho} \geq \phi$ and $A < B/b$, the expression in square brackets (equivalent to $-D_t(0;\lambda)$ in the proof of Lemma A1) is monotonally decreasing and there is a unique value of λ (denoted with λ' in the aforementioned proof) that zeroes it. Thus $M(\lambda;\lambda_0)$ reaches a maximum at $\lambda^* = \lambda'$. **QED**

The optimal choice λ^* corresponds to a resentment state at which, if corruption activity ceased for an infinitesimal time lenght, there would be no gain nor loss in the briber's instant payoff when she reactivates bribing. Note that λ^* is independent from λ_0, agreeing with the assumption of a stationary solution.

But what if the cyclic behaviour depicted by Proposition 1, while possible, is overrun by the alternative, namely starting bribing at some point in time and not stopping from then on? Lemma A2 in the Appendix shows that the alternative cannot yield higher present value of briber's payoffs.

Sensitivity to small increase in parameters

Comparative statics shows that the model behaves sensibly under variations of the baseline parameters. Here is a list of the effects of an increase of either one of the considered parameters:

φ: It does not alter the choice of λ^*. However it reduces the time lenght of each corruption activity period due to accelerated resentment. This has a negative effect on the briber's payoff through an increase of Ψ.

ϕ: It decreases λ^*. Its effect on briber's payoff is positive in all flanks: 1) through a decrease of λ^*, which increases $\gamma(\lambda)$; 2) through an increase of $\left(\frac{\lambda}{\lambda_0}\right)^{\frac{\tilde{\rho}}{\phi}}$ since $\lambda^* < \lambda_0$; 3) through a decrease of Ψ.

$\tilde{\rho}$: It increases λ^*. Its effect on briber's payoff is negative.

When bribing is always instantly beneficial to the briber: a comment

If we get rid of the assumption $A < B/b$, we actually obtain a dramatically different result. Following the proof of Lemma A1, $A \geq B/b$ gives inexistence of a solution for the equation provided by Lemma 1, unless $t = t'$. That is,

Proposition 2: If $\tilde{\rho} \geq \phi$ and $A \geq B/b$, the optimal schedule would consist on being inactive until some time T', and then being active from then on without further interruption.

Proof (Sketch): By Lemma A1 in the Appendix, if the briber stops bribing activity at some moment t, it is never optimal to retake activity ever more. But then, given that $A \geq B/b$ ensures postivie instant payoffs to an active briber no matter the level of resentment λ, it is suboptimal to stop activity at all, once it has been initiated. The remaining question is when to initiate it for the first and only time, that is, finding the optimal T'.

5 Conclusion and possible extensions

We have introduced a model that explains potential cyclicity in corruption through time as a result of voters' cummulative resentment against it. Resentment increases the risk of political turnover in case of corruption detection, hence making bribing more expensive. If there is a resentment level at which the briber does not extract instant positive payoff when bribing the ruling party, it turns out that the optimal schedule of bribing involves cycles alternating inactivity with corruption activity. The model lies on solid literature basis, for instance when calculating optimal bribes: it utilizes the Shapiro and Stiglitz (1984) approach to shirking (corruption in our paper) in the labor market (here the political arena).

The main assumption of the model is that the briber's discount rate is higher than the rate of forgiveness, that is, the rate at which the electorates reduces its resentment during a period of no corruption. We do not have a conclusive mathematical analysis when such assumption is violated. We can however postulate an intuitive hipothesis: when the inequality is reverted by a sufficiently large amount, optimal bribing dynamics may not show cyclicity. That is, the briber is sufficiently patient to wait until resentment is negligible enough, then to engage into bribing activity from then on.

Among the many possible modifications of the model, there is a particularly interesting variation that deserves a complete analysis in further research. We refer to the possibility that resentment could have an impact not only on the chances of political turnover, but also on the bipartisan system itself. At the end, resentment cannot be diminished if an electoral turnover does not have a lowering impact on corruption activity. It may well be that electoral turnout could substantially diminish, giving rise to electoral space for third parties. This lack of differences in the attitude of moderate parties with respect to corruption could explain in part the trend towards instability of bipartidism in modern democracies. It may be in the interest of a rational briber to abide by some risk of a fall of bipartidism.

References

[1] Bicchieri C. and Duffy J. (1997) "Corruption cycles", *Political Studies* XLV, 477-495.

[2] Feitchinger G. and Wirl F. (1994) "On the stability and potential cyclicity of corruption in governments subject to popularity constraints", *Mathematical Social Sciences* 28, 113-131.

[3] Dawid H. and Feichtinger G. (1996) "On the Persistence of Corruption", *Journal of Economics* 64, 177-193.

[4] Rinaldi S., Feichtinger G. and Wirl F. (1998) "Corruption dynamics in democratic societies", *Complexity* 3, 3-64.

[5] Joung-Hun Lee J-H., Iwasa Y., Dieckmannd U. and Sigmund K. (2019) "Social evolution leads to persistent corruption", *PNAS* 116, 13276–13281.

[6] Figueroa V. (2020) "Political Corruption Cycles: High-Frequency Evidence from Argentina's Notebooks Scandal", *Comparative Political Studies* 54, 482–517.

[7] Cooper J. (2021), "Political Corruption Cycles in Democracies and Autocracies: Evidence from Micro-data on Extortion in West Africa", *Quarterly Journal of Political Science* 16, 285-323.

6 Appendix

Lemma A1: Let $\tilde{\rho} \geq \phi$. For each value of λ, there is at most one value $t_\lambda > 0$ such that

$$\gamma(\lambda) = e^{-\tilde{\rho} t_\lambda} \gamma(\lambda e^{-\phi t_\lambda})$$

Moreover, if $A < B\frac{\lambda}{a+b\lambda}$, there is a unique value λ' for which a unique solution to the previous equation exists if and only if $\lambda > \lambda'$. On the contrary, if $A \geq B\frac{\lambda}{a+b\lambda}$, no λ has an associated solution $t_\lambda > 0$.

Proof: Define $D(t;\lambda) \equiv e^{-\tilde{\rho} t} \gamma(\lambda e^{-\phi t})$. Since $\gamma(\cdot)$ is bounded above by A, we have $\lim_{t\to\infty} D(t;\lambda) = 0$. The derivative is

$$\begin{aligned} D_t(t;\lambda) &= -\tilde{\rho} D(t;\lambda) - \phi\lambda e^{-(\tilde{\rho}+\phi)t} \gamma'(\lambda e^{-\phi t}) \\ &= -\tilde{\rho} D(t;\lambda) + aB\phi\lambda \frac{e^{-(\tilde{\rho}+\phi)t}}{(a+b\lambda e^{-\phi t})^2} \end{aligned}$$

When $D_t(t;\lambda) = 0$ the second derivative is negative if

$$
\begin{aligned}
(\tilde{\rho}+\phi)e^{-(\tilde{\rho}+\phi)t}(a+b\lambda e^{-\phi t})^2 &> 2(a+b\lambda e^{-\phi t})\phi b\lambda e^{-\phi t}e^{-(\tilde{\rho}+\phi)t}, \text{ or} \\
(\tilde{\rho}+\phi)(a+b\lambda e^{-\phi t}) &> 2\phi b\lambda e^{-\phi t}, \text{ or} \\
\tilde{\rho}(a+b\lambda e^{-\phi t}) &> \phi(a-b\lambda e^{-\phi t})
\end{aligned}
$$

for which $\tilde{\rho} \geq \phi$ is sufficient. Therefore, $\tilde{\rho} \geq \phi$ implies that $D(\cdot;\lambda)$ has a unique local maximum. Since $\lim_{t\to\infty} D(t;\lambda) = 0$, we have that, *if it exists*, there is a unique point $t_\lambda > 0$ such that $D(t;\lambda) = \gamma(\lambda)$.

For the second statement. Note

$$
\begin{aligned}
D_t(0;\lambda) &= -\tilde{\rho}\left[A - B\frac{\lambda}{a+b\lambda}\right] + aB\phi\frac{\lambda}{(a+b\lambda)^2} \\
&= B\lambda\left[\frac{\tilde{\rho}}{a+b\lambda} + \frac{a\phi}{(a+b\lambda)^2}\right] - \tilde{\rho}A
\end{aligned}
$$

Let $z \equiv \frac{1}{a+b\lambda}$ (and thus $\lambda = \frac{1}{bz} - \frac{a}{b}$) and let $C(z) \equiv \left(\frac{1}{bz} - \frac{a}{b}\right)\left(\tilde{\rho}z + a\phi z^2\right)$. If $C(z)$ is monotonally decreasing, then $D_t(0;\lambda)$ is monotonally increasing in λ. But

$$
\begin{aligned}
C'(z) &= \frac{-1}{bz^2}\left(\tilde{\rho}z + a\phi z^2\right) + \left(\frac{1}{bz} - \frac{a}{b}\right)(\tilde{\rho} + 2a\phi z) \\
&= \frac{-\tilde{\rho}}{bz} - \frac{a\phi}{b} + \frac{\tilde{\rho}}{bz} + 2\frac{a\phi}{b} - \frac{a\tilde{\rho}}{b} - \frac{2a^2\phi}{b}z \\
&= \frac{a(\phi-\tilde{\rho})}{b} - \frac{2a^2\phi}{b}z < 0
\end{aligned}
$$

since $\phi \leq \tilde{\rho}$ and $z > 0$. Thus $D_t(0;\lambda)$ is monotonally increasing in λ. Note that $\lim_{\lambda\to\infty} D_t(0;\lambda) > 0$ iff $A < B/b$. (Incidentally, this proves the last statement of the Lemma: inexistence of a solution when $A \geq B/b$). We conclude that there is a cutoff value λ' for which $D_t(0;\lambda) > 0$ iff $\lambda > \lambda'$, provided $A < B/b$.

If $\lambda \leq \lambda'$, function $D(\cdot;\lambda)$ always decreases. ($D_t(t';\lambda) = 0$ at some $t' > 0$ implies a local maximum at t' as we have seen; but then we reach a contradiction with $D_t(0;\lambda) \leq 0$ and $\lim_{t\to\infty} D(t;\lambda) = 0$.) Therefore there is no $t_\lambda > 0$ solving $\gamma(\lambda) = e^{-\tilde{\rho}t_\lambda}\gamma(\lambda e^{-\phi t_\lambda})$.

If $\lambda \leq \lambda'$, $D_t(0;\lambda) > 0$ jointly with $\lim_{t\to\infty} D(t;\lambda) = 0$ establishes the existence of a solution $t_\lambda > 0$ for the equation $\gamma(\lambda) = e^{-\tilde{\rho}t_\lambda}\gamma(\lambda e^{-\phi t_\lambda})$ via the Intermediate Value Theorem. Moreover we have seen that this value $t_\lambda > 0$ is unique. **QED**

Lemma A2: Suppose $\rho \geq \phi$ and $A < B/b$. Take the (alternative) schedule of being inactive from $t = 0$ to $t = T$ and then being active from then on. The latter schedule cannot yield higher present value of payoffs to the briber, as compared to the cyclic solution found by means of Proposition 1.

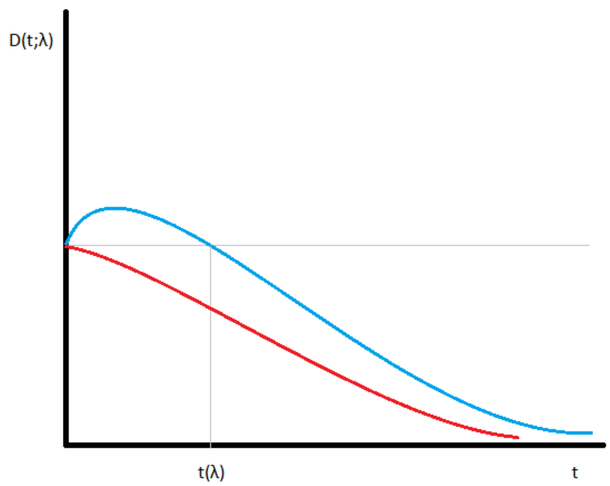

Figure 2: Illustrating the proof of Lemma A1. For λ high enough, $D_t(0;\lambda)$ becomes positive, giving rise to the hump shape of the blue line.

Proof: Consider the alternative schedule above depicted. Using the shortening notation $\lambda(t,T) = \lambda_0 e^{-\phi T+\varphi(t-T)}$, induced present value for the briber is

$$U(T) = \int_T^\infty e^{-\tilde{\rho}t}\gamma(\lambda(t,T))dt$$

Noting that $\lambda_T(t,T) = -\frac{\phi+\varphi}{\varphi}\lambda_t(t,T)$, the first order condition for maximization is

$$U'(T^*) = -e^{-\tilde{\rho}^{T^*}}\gamma(\lambda(T^*,T^*)) - \frac{\phi+\varphi}{\varphi}\int_{T^*}^\infty e^{-\tilde{\rho}t}\gamma_t(\lambda(t,T^*))dt = 0$$

By integration by parts, we have

$$\begin{aligned}\int_{T^*}^\infty e^{-\tilde{\rho}t}\gamma_t(\lambda(t,T^*))dt &= -e^{-\tilde{\rho}T^*}\gamma(\lambda(T^*,T^*)) + \tilde{\rho}\int_{T^*}^\infty e^{-\tilde{\rho}t}\gamma(\lambda(t,T^*))dt \\ &= -e^{-\tilde{\rho}T^*}\gamma(\lambda(T^*,T^*)) + \tilde{\rho}U(T^*)\end{aligned}$$

yielding

$$U(T^*) = e^{-\tilde{\rho}T^*}\frac{\gamma(\lambda_0 e^{-\phi T^*})}{\Psi}$$

where $\Psi \equiv \tilde{\rho}\frac{\phi+\varphi}{\phi}$ as in the main text.

If the first order condition is sufficient, we have

$$U(T^*) \leq \sup_{\lambda>0} e^{-\tilde{\rho}\tau(\lambda;\lambda_0)}\frac{\gamma(\lambda)}{\Psi}$$

with $\tau(\lambda; \lambda_0)$ defined as the unique solution to $\lambda \equiv \lambda_0 e^{-\phi\tau(\lambda;\lambda_0)}$. But note that the right-hand side of the inequality is precisely the optimal value calculated in the main text, through Proposition 1.

Finally, if the first order condition is not sufficient, the remaining candidate is $T^* = 0$. (The other extreme, $T^* = \infty$, implies zero present value.) If such schedule is optimal from $t = 0$ to $t = \infty$ with starting value λ_0, for any $T > 0$, it constitutes an optimal schedule from $t = 0$ to $t = \infty$ with starting value $\lambda_0' = \lambda_0 e^{\varphi T}$. Or, in other words, such schedule is optimal for any starting resentment state λ_0. But this constitutes a contradiction, since $\lim_{\lambda_0 \to \infty} \int_0^\infty e^{-\tilde{\rho}t}\gamma(\lambda_0 e^t)dt < 0$ (since $A < B/b$.) **QED**